Sabbatical Project Report

Spring and Fall 2007

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Comprehensive Statement of Sabbatical Request

Description:

The Learning Assistance Center offers two courses, LERN 48 and LERN 49, that allow students unable to place into pre-algebra (MATH 50) a means by which to practice and refine their basic arithmetical skills. However, in most cases the skills to be practiced are identical to those seen in the fourth to eighth grade standard curriculum, and there is near unanimous agreement that a semester course that merely repeats that curriculum fails to generate much student interest or success. The Learning Assistance faculty agree that introducing students to additional strategies and activities related to learning skills help with students' success, and we cover some learning strategies within our curriculum.

The issue of what strategies best assist students that are not ready for college level mathematics has been taken on by a variety of professional organizations, including the College Reading and Learning Association (CRLA), the National Association for Developmental Education (NADE), and more recently, the American Mathematical Association of Two Year Colleges (AMATYC). It is hoped that a survey of these organizations' literature will be of use in answering the central question behind my request for sabbatical: "How can the LERN 48 and 49 courses be improved?"

The first phase of my sabbatical would be to research suggested practices from NADE, AMATYC, and selected other sources. I would join both organizations' developmental education special interest groups (MathSPiN and AMATYC-Share) and would review the back issues of both organizations' journals for articles related to arithmetic. I would also review similar articles in the Journal of Developmental Education and would solicit suggestions for other resources from members of Mt. SAC's Developmental Education Team. These would be summarized and organized for others and will also form the basis for the later phases of my sabbatical. (Ideally, the result of this first phase should be equivalent to analyzing a packet of supplemental readings intended for students taking a graduate level course on the teaching of arithmetic in a developmental manner to adult students. However, I do not believe that any such course exists.)

The second phase of my sabbatical would be to study recent materials on mathematical study skills and mathematical anxiety. Although I am fairly well versed in these subjects, there have been many recent books published in this field by the likes of Paul Nolting, Cynthia Arem, and Shelia Tobias, and I would like to see what new ideas these authors offer and how closely their material relates to the short essays I have written for my students. LERN classes already embed study skills within their course outlines, but over time the traditional study skills such as note taking have been joined by more affective concepts such as emotional preparedness, and I would like to observe if there is any consensus on the importance of the affective domain in the mathematics classroom. My findings would again be summarized for others and would likely cause me to modify my short essays during phase five of the sabbatical.

The third phase of my sabbatical would be to study selected textbooks. A dozen possible texts exist for LERN 48 and 49, but those adopted recently by the department and those written by professors who teach equivalent classes (e.g., Bello, Staszkow) would be given particular attention with regard to how they approach mathematical topics in a non-traditional manner. Also part of this phase would be the study of our current MATH 50 text (Pre-Algebra by Carson, 2nd edition). This phase would not be summarized, but gaps between the LERN 49 course outline and the Carson text would be noted and brought back to the Learning Assistance Department, and the notes taken during this phase would be used during phase five of the sabbatical.

Phase four of my sabbatical would be to write several supplemental units for the LERN 48 and 49 classes that would be independent of any particular adopted text. Certain materials are clearly needed. First of all, both LERN 48 and LERN 49 cover factoring, prime numbers, prime factorization, and least common multiple. Although our current practice of using the same text for LERN 48 and 49 reduces costs to our students, it also means that students who take both classes begin LERN 49 repeating the same exercises they just practiced in LERN 48. Secondly, the topics of area, perimeter, and arithmetic mean need to be given their own supplemental unit. In most texts (including our current one) these topics are covered in excessive detail in a chapter (or chapters) that contain topics we do not require. Thirdly, the topic of integers needs to be given a supplemental unit so that it can be introduced earlier in the semester. Most texts (including our current one) leave integers until late in the semester while including signed decimals and fractions. This inclusion of decimal and fraction review makes the concept and basic workings of integers needlessly complicated. It may be that the third phase of my sabbatical reveals additional areas that require a supplemental unit, but it is not my intention to write a complete textbook. All written materials would be made freely available in an editable form.

The final and fifth phase of my sabbatical would be to write a notebook for LERN 48 and 49 students. Although note taking, time management, vocabulary, and organization are covered to some degree by all LERN 48 and 49 instructors, I have come to believe that many students in LERN classes do not enter their classes with adequate note taking skills. Therefore, I intend to create a notebook that organizes the note taking process. Most educational objectives in the course (there are roughly 50 such objectives in LERN 48 and 100 in LERN 49) will have a two page layout. The left page will be modeled after "Cliff's Notes" or "Math for Dummies" in that the page will contain very brief summaries of procedures along with helpful tips gleaned from my existing notes or phases 1 – 3 of the sabbatical project. The right page will contain blank space for the student's own notes and comments along with tips for taking notes, prompts for writing down vocabulary terms and/or "warm-up" exercises. ("Warm-up" exercises are relatively easy exercises intended to be done quickly so that a peer or instructor can verify that the student understands the process at hand.) The notebook would also contain monthly and weekly time management schedules, a glossary of mathematical terms, appendices of collected cooperative and manipulative-based activities, and a collection of one page essays with their suggested writing prompts. Since the notebook would again be freely available in an editable form, it could be edited by an individual instructor to contain other handouts and worksheets as well.

Timeline

Note: Timeline changed for Spring/Fall 2007 at dean's request.

February – Phase I

- Order online access to developmental journals
- Locate college libraries with hard copies of developmental journals (likely UCD)
- Read archives of special interest groups

March – Phase I, Phase II

- Read and summarize developmental journal articles
- Inquire to special interest groups of suggested additional data sources
- Read and summarize suggested additional data sources
- Organize summaries and copies of articles/data into binders for department use
- Acquire texts on study skills, mathematical study skills, and math anxiety

April – Phase II

- Read acquired texts on study skills, mathematical study skills, and math anxiety
- Cross reference acquired texts for use of technique and perceived importance of affective domain
- Write summary of findings for Phase II

May – Phase III

- Read Bello, Bittinger, Statzkow, and VanDyne fundamental math textbooks noting scope, sequence, non-traditional treatments, and collaborative & manipulative-based activities.
- Read Carson and CPM pre-algebra texts, noting scope and treatment

June – Phase IV

- Write supplemental materials for primes, factors, L.C.M. and related objectives
- Stop work at end of spring semester for vacation

July – none

August – Phase IV

Begin work in late August at start of fall semester

Write supplemental materials for area, perimeter, and mean

September – Phase IV, Phase V

Write supplemental materials for integers

Learn Adobe InDesign well enough to use for layout of notebook

Outline notebook layout and write master pages

October – Phase V

Write notebook pages for LERN 48 objectives

Start notebook pages for LERN 49 objectives

November – Phase V

Finish notebook pages for LERN 49 objectives

Select or create supplemental materials for notebook

December – Phase V

Complete notebooks

Statement of Value and Benefits

On a personal level, this sabbatical would enable me to become current with the topics that directly relate to my everyday work. I read some professional journals and go to workshops and conferences, but there is never enough time to read all the published information, analyze the provided material, or often even to reflect on the workshops I have attended. At the conclusion of this sabbatical, I should be considerably better informed as to the state of developmental instruction in arithmetic, and the process of creating a workbook should provide many hours of contemplation about why I teach each objective the way that I do. In addition, over the past three years I have served as a reviewer for three textbooks and the iLrn website, and being given the time to place what I have learned about different texts into a concrete project would be professionally and personally rewarding.

On a departmental level, this sabbatical would enable me to create some much needed materials for free use by our faculty. I have solicited the input of every full-time and several part-time LERN math instructors, and I am confident that the projects noted in Phases IV and V of my sabbatical request would gain use within the department. Furthermore, the workbook would be a valuable resource to the part-time faculty who are expected to teach over 2/3 of our sections for the next few years.

On a collegial level, this sabbatical would provide me with material that could be presented at workshops and conferences. This year's CRLA and AMAYTC conferences had very few workshops relating to arithmetic and the developmental student, but those workshops that did exist were extremely popular. Mt. SAC's Learning Assistance Center is already well regarded by many, and I believe that several interesting conference workshops could result from this new material which could also increase the college's visibility as an innovator. An improvement in the materials we use should also measurably enhance students' success.

Abstract

This sabbatical is a year long project that seeks to summarize and place into practice recent research in the field of developmental mathematics as it affects the LERN math classes. The sabbatical will begin with the review of research articles on developmental mathematics, current textbooks relevant to LERN math and pre-algebra, and texts on mathematical study skills and math anxiety. Information gleaned from such review will then be used to write supplemental materials and a guided notebook for students to use while taking LERN math courses. All created material will be provided in editable form to all interested instructors.

Statement of Purpose

This sabbatical provided the author the time necessary to review recent research journals, to compare currently available mathematical study skills and subject matter texts, and to create supplemental materials relevant to the LERN mathematics classes.

Most arithmetic materials are written with children in mind, and thus simple Internet searches are not useful when attempting to locate what other faculty are doing with their adult learners. Consequently, keeping current in the field is a time consuming task, particularly since no professional organization specializes in developmental mathematics. A purpose of the sabbatical was to read the professionally edited articles that were scattered amongst various journals relating to mathematics, developmental education, and educational philosophy.

Commercially available texts exist relating to mathematics anxiety, mathematical study skills, and arithmetic, and these vary from thin pamphlets on a single topic to comprehensive textbooks. To my knowledge, no one I knew had ever taken the time to thoroughly read more than a few of these texts, and I did not know to what extent these "experts" agreed should be the scope of mathematical study techniques. A purpose of the sabbatical was to read texts marketed to LERN mathematics students in order to develop a sense of to what extent uniformity exists and to create helpful materials in the field of mathematical study techniques.

LERN 48 and 49 normally share the same textbook, so where the two courses overlap, students are faced with having exactly the same pages to read and problems to work in each course. A purpose of the sabbatical was to create appropriate materials so that LERN 48 and 49 students could have disjointed assignments regardless of the text in use.

Preface

When I started work on my sabbatical in February 2007 I did not anticipate the end result. My initial idea for a sabbatical was to write a math textbook for the LERN math classes. This of course implied that I knew what I was doing, and to some extent this was true. I had studied Jean Piaget and read the works of Lev Vygotsky and Malcolm Knowles, so I felt confident that I understood the needs of adult learners and could create a developmentally appropriate course. Howard Gardner's work in the 1980's was well known to me, and I felt sure that I could find or create materials that dealt with the various human intelligences. My experience with reviewing textbooks and editing some of the commercial on-line materials made me confident that I knew the typical scope and sequence of an arithmetic text. Everything was in order except that I knew very little of the research that had taken place in the last ten years. At the urging of my department chair I modified my original proposal to develop a sabbatical application that was part research and part product. In short, I hoped to learn what had occurred in the past decade after which I would try to put it into practice.

It did not take long to narrow my scope of study to the United States. Although there are over fifty journals that in some way deal with educational development, mathematics, or both, only a few organizations were able to help with my particular set of circumstances. In the first place, the entire concept of developmental education for adults seems to be lost outside of the United States. Adult education in Great Britain, Canada, and Australia appears to focus on obtaining the necessary skill set for life, not academia. Consequently, although the materials available online from outside the United States are considerable, nearly all fall in the "consumer math" category. It is unclear to me how a person over the age of 18 can ever hope to learn introductory algebra in some countries without the hiring of a private tutor.

The organizations within the United States that deal with developmental mathematics generally fall into two categories: those that deal with development, and those that deal with mathematics. Representing the developmental side, I selected the College Reading and Learning Association (CRLA) and the National Association for Developmental Education (NADE). Both groups publish a journal, and between my own personal copies, online access, borrowing, and a few stops at college libraries I was able to locate and to read nearly every issue published in the last seven years. Many of the articles did not relate to mathematics, but the two volume "Best Practices in Developmental Mathematics" published by the Mathematics Special Professional Interest Network arm of NADE provided much helpful information, and I highly recommend the perusal of these documents.

The mathematical side was represented primarily by education groups: the California Mathematics Council (CMC), the American Mathematics Association of Two Year Colleges (AMATYC), and the National Council of Teachers of Mathematics (NCTM). Both CMC and NCTM deal with P - 14 courses, so while their periodicals and web sites held much in the way of teaching mathematics, only a small portion applied to adult learners. Once again, a single treasure trove revealed itself: Beyond Crossroads -- AMATYC's guide to implementing mathematics standards in the community college.

February and March were spent browsing web sites and skimming the following volumes (many of which are available on the organization's web sites):

CMC Communicator -- Dec. 2000 - Sep. 2006
NCTM Mathematics Teacher -- Sep. 2002 - Jan. 2007
NCTM Mathematics Teaching in the Middle School -- Sep. 2003 - Jan. 2007
NCTM Journal for Research in Mathematics Education -- May 1999 - Jan. 2007
The AMATYC Review -- Spring 2002 - Fall 2006
CRLA Journal of Teaching and Learning -- Spring 2001 - Fall 2006
NADE Monographs -- 1999 - 2003
NADE Digest -- Fall 2005 - Spring 2006
NADE Math-SPIN Newsletters -- 1997 - 2007

While surfing, I stumbled upon two web sites created by professors just returning from sabbatical themselves. Ted Panitz (of Cape Cod Community College) and Jack Rotman (of Lansing Community College) proved to be wonderful sources of information. Also about this time, I was forwarded a copy of the just released "Basic Skills as a Foundation for Student Success in California Community Colleges" by the Center for Student Success. It is worth noting that had my sabbatical fallen a year earlier, none of the last three mentioned sources nor the AMATYC document would have been available. The issue of teaching developmental mathematics to adults is suddenly gaining the attention it requires which pleases me.

I would be more pleased if there were some consensus on what exactly to do. Education has been likened to the medicine of a hundred years ago, when bringing the same symptoms to a dozen different doctors would yield a dozen different prescriptions. Right now educational psychology is rife with experiments involving low sample sizes, inadequate controls, and the tendency to confuse correlation with causation. Although I eventually ceased to be amazed by the low quality of the research being published, I am convinced that many of the assumptions we hold dear may in fact not be based on any facts. As a result, the first part of the sabbatical did not significantly influence the materials I was to write some months later.

Although I continued to explore articles on the Internet for the rest of the year, most of April and May was spent reading a selection of arithmetic textbooks and study skill books appropriate for the LERN mathematics student. Tables were constructed relating to the scope and sequence of each textbook, although quite frankly there aren't many differences between them. A few introduce negative numbers towards the front of the book instead of the rear, but otherwise the same topics are found in them all. The quality of the problem sets does vary however, with some focusing solely on skills development while others have well thought out application and open-ended problems. As for the study skill books, although they do not generally contradict each other, they do vary considerably regarding the topics they find important enough to include. I have chosen to include a series of "book reports" on these texts, which appear later in this report.

After a pleasant summer break, late August through mid-October were spent creating replacement chapters for where the LERN 48 and 49 courses overlapped. I worked on a philosophy that assumed that LERN mathematics students did not generally read their textbooks -- an assumption I may safely make due to much personal observation of its truth. Consequently, I endeavored to create as brief a document as possible. While the information presented might well be enough to trigger a student's memory, the intent is that these pages supplement a good teacher and the time spent in class. This is significantly different from the approach taken by most books, which attempt (usually unsuccessfully) to create an entire self-study course for the student.

My "texts" are contained in this sabbatical on blue paper, and despite their brevity took quite some time to prepare. I generally had only space for one method of solution, so I generally chose techniques that lent themselves to a class discussion. While formulas are provided, my hope is that an instructor will use the my work as a means to introduce the concepts involved, saving the mechanics for mentioning later. These materials come with their own worksheets (contained in this sabbatical on green paper) which focus on harder (group) and conceptual questions first so that these issues are dealt with in class or while the student is attending lab. The worksheets end with exercises selected from the previous four or five topics. This "spiral review" isn't used by most of the currently available texts, but the technique is recommended by many publishers of K-12 arithmetic materials, including CPM and Saxon.

Mid-October through the end of the sabbatical were spent creating notebooks for LERN mathematics students. Part of the LERN mathematics courses involve the teaching of note taking, but my experience was that as much as one might endevor to explain the mechanics and value of taking notes, a majority of students simply copy what the instructor places on the board. Therefore, I decided to eliminate that crutch. After some thought, I came up with a format that provides students with most of the text that I anticipate the majority of LERN 48 and 49 instructors to place on the board. Most, but not all. By providing students with partial notes, I hope to be able to focus students' attention on why they are taking notes and how to discern whether an item under discussion is worth including in their notes. I also expect to spend less class time watching them copy.

Concerns about the notebooks' length (82 pages for LERN 48 and 100 pages for LERN

49) made me reject my original intention to include a wide variety of projects and writing prompts within the notebooks. I did include examples of both, however, and trust that my fellow professors will modify the notebooks to meet their needs. A binder of interesting materials will be made available in my department, although I have not reproduced the pages here.

Phase I – Suggested Developmental Practices

Developmental educational literature shows the relative youth of the field. Developmental education as a field of study only dates to the 1970s, as evidenced by the establishment dates of its most influential journals. The Journal of Developmental Education, published by the National Center for Developmental Education (NCDE) dates to 1978, and NCDE itself was established in 1976. Similarly, the National Association for Developmental Education (NADE) was established in 1976, and the College Reading and Learning Association (CRLA) only formally incorporated "learning" into its name in 1979, although the association itself was founded as the Western College Reading Association in 1966 (Stephens). Perhaps as a consequence of this youthfulness, the articles published often appear to be scientifically immature. I frequently read papers that used sample sizes far too small to be valid, utilized inappropriate (or no) statistical tests, or stated statistical significance without providing any of the original data. Worse, there are numerous papers that reference these flawed papers as if they are established fact. If nothing else, this phase of my sabbatical has caused me to note that the field of Developmental Education is currently wanting of large-scale scientifically valid research projects.

Nonetheless, one does tend to develop a sense that certain concepts or patterns routinely come up while reading the literature. To what extent this is blind regurgitation I cannot tell, but in the absence of better data I will proceed as though the trends in the literature over the last decade are in fact valid observations.

The first phase of my sabbatical was to research suggested practices from NADE, the American Mathematical Association of Two Year Colleges (AMATYC), and selected other sources. I joined both organizations developmental education special interest groups. However, neither group was particularly active, and over the first month a total of three posts were made to the groups e-mail lists. Fortunately, the NADE special interest group (MathSPIN) has an archive of its activity on Yahoo Groups, and consequently I was able to read several years of past posts. Many of these past posts provided external links (web sites and online articles) that I then read. I also skimmed each issue of a number of journals and periodicals published since 1997, including the AMATYC Review and the Journal of Developmental Education, and read any papers within that appeared to might possibly apply to arithmetic-level mathematics. As such, well over a hundred papers were read, but in the interest of having my sabbatical report actually be of value to someone, only the fifteen most interesting or relevant papers are summarized below. I would encourage my colleagues to read these articles.

In addition, several other items stand out as worthy to mention to my colleagues. First of all, both NADE and AMATYC have published large-scale documents in recent years. NADE published two volumes of Best Practices in Developmental Mathematics (in 2002 & 2003) that are essentially compilations of suggestions provided by educators in the field. In addition, AMATYC just recently published Beyond Crossroads (in 2006), which is a comprehensive document of mathematics standards for two year colleges. Finally, the California Community College System Office has just recently funded a survey of developmental education literature entitled Basic Skills As a Foundation for Student Success in California Community Colleges (in 2007). All are well worth the time to read.

There is some agreement that developmental students are not all alike. Hardin has categorized developmental students into no fewer than seven categories, including two (those in college for nonacademic reasons and those with extreme learning problems) who are unlikely to benefit from developmental classes (Hardin). The remaining categories all involve some combination of insufficient preparation, learning disabilities, or a lack of background skills necessary for success. Anxiety also appears to be a common issue amongst developmental students, and while some consider anxiety control a background skill, others consider anxiety to be an environmental factor largely created by the instructor (Tobias). Thus I will present my findings in my search for suggested practices in these four categories: student preparation, disability accommodation, background skill acquisition, and environmental factor control.

Student Preparation

Many developmental students are in developmental classes either because they previously made choices inconsistent with getting a college education or because their previous choices did not adequately prepare them for their current career goals (Hardin). Frequently these students do not realize their weakness in mathematics, and generally require a mandatory assessment process to discover their need for a developmental class (Hunt), although a few colleges have reported success with carefully crafted self-assessment processes (Barr).

Flexibility seems to be key to a program's success. Ted Panitz's survey of community colleges in Massachusetts found that additional unit classes and multiple options for classes (including modular, accelerated, and self-paced) improved overall student success. Panitz also found that certain enhancements to the traditional class benefited students, such as mandatory tutoring by trained tutors and early warning advisement systems. (Panitz). These enhancements are also mentioned by others (Hunt, Vasquez).

As for the class itself, few now promote providing a simple review of material. However, there are a multitude of suggestions as how to best accomplish the learning of developmental mathematics. Selina Vasquez notes that the curriculum should include "both fundamental and problem-solving skills" and that one should "base examples, activities, and problems on real-world content" (Vasquez, pp. 20, 21). The AMATYC Standards cite the need for "active learning", and suggest a variety of strategies including those that can be considered cooperative, collaborative, discovery-based, interactive, question-posing, and writing-based (AMATYC, p. 54). As an alternative, the researchers Krank and Moon recommend a combination of cooperative assignments and mastery testing (Krank and Moon). Some authors proffer the constructivist style (where students construct their methods of problem solving based on their prior knowledge) as exemplary (Bakal), while others warn that it may be inappropriate to assume that the philosophy behind constructivism (which is based upon the research of Piaget and Vygotsky, both of whom were interested in the development of children) necessarily applies to adults (Anderson, Rotman).

Perhaps the latest philosophies of what is often called "Brain-based Research" can be drawn upon to help clear up the issue. Although it is a considerable leap between education and neuroscience, the concept that students have to be considered holistically (including physically and emotionally) is gaining credence (Di Muro). "Brain-based" philosophies include the ideas that we remember better when associations to our prior knowledge and patterns are exposed, and we tend not to learn well if the concepts to be learned are taught out of context (Laughbaum). These ideas suggest to me that a variety of teaching techniques similar to those that students have experienced before and real-world problems based on experiences that students have had before are perhaps the most likely to bring success. In any event, it is important not to overlook the training of the instructors and the encouragement of their sharing tips and ideas (Vasquez).

Disability Accommodation

A number of developmental students have known disabilities. However, there are many developmental students with undiagnosed learning disabilities (Hardin), and so both student and instructor are essentially in the dark when it comes to helping these students. In addition, since even those without a learning disability often have underdeveloped learning capabilities, several authors recommend using a variety of instructional techniques and providing alternative assignments and means of evaluation (Poindexter, Vasquez).

Background Skill Acquisition

College professors frequently demand prerequisite skill sets from their students regardless of whether the class has any prerequisite classes. These skills include having sufficient language, interpersonal, and study skills to function in the class. Recently, much of the attention appears to be given to study skills, and there is evidence that the teaching of skills such as organization and note taking can both raise test scores and student confidence (Lewis and Clark). The teaching of study skills is becoming so accepted that now AMATYC is calling for "substantial changes" to the developmental curriculum; one of these changes being to "develop students' study skills and workplace skills to enable them to be successful in other courses and in their careers" (AMATYC, p. 41). Hence, it is clear that mathematics teachers should also be teaching study and workplace skills. One of the more useful workplace skills is the ability to work in a cooperative group. This is not an innate ability, as Hoek, et. al. found benefits after explicitly teaching students how to work in such groups (Hoek).

Of more questionable value are meta-mathematical skills (the analysis of one's own learning of mathematics) and reasoning skills. Although meta-mathematical skills may seem valuable, Adibnia and Putt found that lessons based on improving such skills did not create a gain on posttest scores (Adibnia and Putt). Reasoning skills (also called critical-thinking skills) are taught in most good developmental education programs (Boylan), yet there is a tendency for students to use their personal experience far more than their reasoning skills when analyzing non-routine problems (Lithner), and that students tend to hold on to their misconceptions (about proportional reasoning) even when confronted with strong evidence that their misconceptions were incorrect (Van Dooren). From this I am forced to wonder whether students who learn meta-mathematical and reasoning skills in class remember to actually apply those skills to their work.

Environmental Factor Control

Student comfort and contentment are seen as important goals in the developmental classroom. One way to build such contentment is to create a sense of belonging such as noted by the authors who recommend the practice of forming learning communities (Panitz). Another way is through the reduction of anxiety. Math anxiety has become so prominent an issue that learning to control it is cited by AMATYC as one of the three expectations they have of developmental students, and that "knowledge of mathematical anxiety and [its] associated coping strategies" is necessary for those teaching developmental mathematics courses (AMATYC, pp. 42, 64). Yet anxiety is not all bad. A multinational study by H. Ho et. al. found that math anxiety can be categorized into affective (feeling scared, anxious, or dreading doing math) and cognitive (worrying about not doing well in math in the future or about falling behind in math) categories. Although affective anxiety correlates negatively to success, cognitive anxiety does not (Ho, et. al.).

Yet developmental students do not enter our mathematics classrooms without past experience in mathematics classrooms, and those past experiences are often not positive. Although she was speaking of underprepared intermediate grade children, Katy Early's description of underprepared students as having spent insufficient time doing mathematics, as struggling in mathematics for several years, as having convinced themselves that they can't succeed in math, as having learned to avoid math as a boring, repetitive, unpleasant subject, and as having had little opportunity to discuss or make sense of mathematics probably describes many adult developmental students (Early). As such, "we might find it more profitable to address issues of sense-making and attitude than to impose repeated calculation practice" (Early, p. 38). This call to reduce calculation practice has also been repeated by others (Ralston).

As we endevor to create courses that meet our students' needs, it is also important to note that we need to be entertaining enough so as to hold their interest. As Ed Laughbaum notes, "Students cannot learn if they are not paying attention. The main ingredient for getting attention is novelty" (Laughbaum). In my opinion, the idea of "novelty" implies the creation of an enriched environment which has not just a set of activities for the students, but has enough variation amongst these activities so that the students perceive them as being distinct from each other. In short, we want to create a classroom where "everyone should be having fun" (Vasquez, p. 22).

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Rotman, J., "Constructivism and Telling", online at http://www.lcc.edu/~rotmanj/sabbatical2006/ 7%20Constructivism%20and%20Telling.pdf, written Dec. 2006 and viewed on 18 May 2007.

Stephens, D., "Pathways Through History: Unprepared Students Go To College" NADE Monograph 2003: 15 - 37, reprinted online at http://http://www.nade.net/publications/ monograph.html, viewed on 22 March 2007.

Tobias, S., "Environmental Barriers to Student Success", in T. Armington, ed., "Best Practices in Developmental Education, v. 1": 3, online at http://www.etsu.edu/devstudy/ spin/ bestpractices.pdf, viewed 6 June 2007.

Van Dooren, W., et. al, "Improper Applications of Proportional Reasoning." Mathematics Teaching in the Middle School 9 (Dec. 2003): 204 - 209.

Vasquez, S., "Reforming the Developmental Mathematics Classroom", in T. Armington, ed., "Best Practices in Developmental Education, v. 1": 20 - 22, online at http: //www.etsu.edu/devstudy/spin/bestpractices.pdf, viewed 6 June 2007.)

Research Article Abstracts (Suggested Reading):

Hoek, Terwel, and Van den Eeden (1997) found that there are definite benefits to teaching students how to use strategies in solving mathematical problems and how to work effectively in cooperative groups. (Hoek, D., et. al., "Effects of Training in the Use of Social and Cognitive Strategies: An Intervention Study in Secondary Mathematics in Co-Operative Groups" Educational Research and Evaluation 3 (Dec. 1997): 364 - 389)

Hallam and Price (1998) found that introducing background music into a classroom of children with emotional and behavioral difficulties improved mathematics performance (Hallam, S., and Price, J. "Can the Use of Background Music Improve the Behaviour and Academic Performance of Children with Emotional and Behavioural Difficulties?" British Journal of Special Education 25 (Jun. 1998): 88 - 91)

Adibnia and Putt (1998) discovered that lessons involving a variety of cooperative process problems that focused on developing middle school students' awareness of their own thinking did not create a gain of post-test scores significantly different from the control group. (Adibnia, A. and Putt, I., "Teaching Problem Solving to Year 6 Students: A New Approach." Mathematics Education Research Journal 10 (Dec. 1998): 42 - 58)

Ralston (1999) discusses the decreasing utility of pencil and paper calculations and argues for a curriculum consisting of estimation skills and technology usage. He notes the near unanimous usage of calculators for algebra and higher classes and amongst professionals who use math is in conspicuous contrast to the traditional K-7 mathematics curriculum. (Ralston, A., "Let's Abolish Pencil-and-Paper Arithmetic" Journal of Computers in Math and Science Teaching 18 (1999): 173 - 194, reprinted online at http://www.doc.ic.ac.uk/~ar9/abolpub.htm, viewed 23 June 2007.)

Boylan (1999) summarized his research into the causal factors of quality developmental education programs and found most good programs:

- result from an institutional commitment to the concept of educational development
- are delivered by well-trained people
- are student-centered and holistic
- connect with the collegiate curriculum
- are well coordinated
- are based on explicit goals and objectives
- incorporate critical [thinking] skills into its activities
- evaluate themselves (Boylan, H., "Developmental Education: Demographics, Outcomes, and Activities" Journal of Developmental Education 23 (Winter 1999): reprinted online at http://www.capecod.edu/faculty/tpanitz/sabbat.pdf, viewed 16 June 2007.)

Lithner (2000) noted that university students tend to use established experience far more than plausible reasoning when analyzing non-routine problems (Lithner, J., "Mathematical Reasoning in Task Solving." Educational Studies in Mathematics 41 (Feb. 2000): 165-190)

Ho, et. al. (2000), in a multi-national study on mathemaics anxiety, found that while affective math anxiety (feeling scared, anxious, or dreading doing math) had a significant negative correlation to mathematics achievent, cognitive math anxiety (worrying about doing well in math or about falling behind in math) had no significant correlation with mathematics achievement. (Ho, H., et. al., "The Affective and Cognitive Dimensions of Math Anxiety: A Cross-National Study" Journal for Research in Mathematics Education 31 (May 2000): 362 - 379)

Young and Ley (2001) described the self-regulatory deficiencies that developmental students tend to have and suggested compensatory teaching guidelines to attempt to correct this. (Young, D., and Ley, K, "Developmental Students Don't Know That They Don't Know Part II: Bridging the Gap." Journal of College Reading and Learning 31 (Spring 2001): 171 - 178)

Krank and Moon (2001) tested mastery and cooperative learning environments and found that both academic and affective outcomes were best achieved through a combination of the two environments, with assignments being done cooperatively and testing being done via a mastery environment. (Krank, H., and Moon, C, "Can a Combined Mastery/Cooperative Learning Environment Positively Impact Undergraduate Academic and Affective Outcomes?" Journal of College Reading and Learning 31 (Spring 2001): 195 - 206)

Anderson, et. al., (2002) reviewed the literature relating to situated learning (the process of learning in a "real-world" environment) and constructivism (the concept that learning is primarily a personal construct that cannot easily be broken down or generalized in a generic fashion) and found that most of the research lacked solid scientific grounding. The authors go on to argue that both abstract and situated learning appear to have merit in certain circumstances, and that many, but perhaps not all instructional situations can be decomposed into generic patterns. (Anderson, J. R., et. al. "Applications and Misapplications of Cognitive Psychology to Mathematics Education" Texas Educational Review (Summer 2002): reprinted online at http: //act-r.psy.cmu.edu/papers/misapplied.html, viewed 18 June 2007.)

Lewis and Clark (2003) embedded study skills into a developmental algebra course, focusing on organization and notetaking activities. Students noted feeling more confident with their note-taking and overall success levels rose by about ten percentage points. Curiously, Lewis and Clark also noted no correlation whatever between students' self-predicted grades and actual grades. (Lewis, R. and Clark, K, "Embedding Study Skills Into a Developmental Algebra Course" AMAYTC Review 25 (Fall 2003): 43 - 56)

Van Dooren, et. al. (2003) noted that 12 and 16 year old students with misconceptions about proportional reasoning tend to maintain their misconceptions even when confronted with strong evidence that their misconceptions were incorrect. (Van Dooren, W., et. al, "Improper Applications of Proportional Reasoning." Mathematics Teaching in the Middle School 9 (Dec. 2003): 204 - 209)

Poindexter (2005) notes that a significant number of developmental students have hidden or undiagnosed learning disabilities and therefore it is in the best tinterests of the instructor to use a variety of instructional strategies. Specific suggestions include varying the activities used during each class, using visual displays, and offering alternative assignments to meet educational goals. Poindexter also suggests using easy to read fonts and colored paper, as black on white is often the most difficult combination to read. (Poindexter, S., "Learning Styles of the Developmental or the Learning Disabled?" NADE Digest 1 (2005): 26 - 28)

Di Muro (2006) notes that many developmental students have previously been unable to grasp the connections between mathematical concepts, relying instead on memorization as a short-term fix. This implies that long-term learning does not occur. Specific suggestions for improving the situation involve connecting new concepts to previously learned ones, using a mix of instuctional methods, asking questions that go beyond the recall level, and encouraging writing assignments related to the mathematical concept at hand. Frequent assessment, rapid feedback, and the use of holistic, conceptual models are also recommended. (Di Muro, P., "Best Practices in Mathematics Education: Teaching for Understanding" NADE Digest 2 (2006): 1 - 8)

Harwell, et. al. (2007) sampled students in seven large school districts and found that NCTM Standards-based curricula do not impede the mathematical performance and development of higher achieveing students, and in some cases the curricula appear to have contributed to better student performance on traditionally oriented standardized tests. However, the authors note that traditional tests may not be the best way to determine performance. (Harwell, M., et. al., "Standards-based Mathematics Curricula and Secondary Students' Performance on Standardized Achievements Tests" Journal for Research in Mathematics Education 38 (Jan. 2007): 71 - 99)

Phase II – Mathematical Study Skills

Historically, the Mt. SAC Learning Assistance Center has focused on the traditional study techniques such as notetaking and time management. In recent years however, affective issues such as anxiety control have become of interest to our department, and the second phase of my sabbatical was to survey selected texts on mathematical study techniques to see if there is a general concensus on which topics are most important. After reading and analyzing six such texts, I must conclude that there does not appear to be much consensus as to what a math study guide should contain.

Affective Topics

Anxiety issues are referred to in the majority of texts covered, but there are some texts that do not consider the concept of anxiety at all. Some books simply note the need to have a "positive approach" (Smith, p. 21). Difficulties on tests are simply "mental blocks" that can be avoided by "solid preparation" (Smith, pp. 101, 103). However, most books deal with math "myths" like "only smart people can do math" (Ooten, p. 18) and devote at least some time to providing an overview of the causes of anxiety. A few texts also deal with the historical aspect of anxiety, for in a number of cases the authors claim that anxiety can be triggered by beliefs formed in the elementary school years. Generally when a historical element is provided, it is meant as a reassurance that the author understands the reader's fears. The point is always that "your past negative math experiences need not continue to burden you." (Arem, p. 25)

Not all books cover ways to control anxiety, but those that do usually discuss physical relaxation techniques or positive visualizations. The visualizations generally involve closing one's eyes and thinking of a positive or relaxing scene. The student is then instructed to "visualize this relaxing scene for one to two minutes." (Nolting, p. 99) Physical relaxation techniques seem to have their focus on breathing exercises, with headings like "The Calming Breath" and "Deep Abdominal Breathing Techniques". (Arem, p. 34)

Attitude control and avoiding negativity are covered in all but one text. Paul Nolting mentions that students have a "locus of control" which can be either internal or external. Those students with an internal locus of control feel that they have the power to change their situation. On the other hand, those with an external locus of control feel that "their lives are controlled by outside forces ... and they can do nothing about their problems." (Nolting, p. 202). Unsurprisingly, Nolting suggests that students with an internal locus of control tend to work more towards earning high math grades. Although the wording varies in other texts, the same ideas are found. Cynthia Arem asks students to have a list of positive affirmations like "I'm capable of learning math" and "I can understand math if I give myself a chance" (Arem, p. 45), while Cheryl Ooten invites students to restate their thoughts in a positive manner, like "I don't understand yet". (Ooten, p. 58)

Overall, the majority of books consider the affective aspects of learning to be important, comprising as much as 25% of one's academic achievement. (Nolting, p. 40) Most of the aspects mentioned deal with either avoiding or managing negative issues (such as anxiety, stress, and frustration) or establishing positive emotions through affirmations and motivation techniques. Enough of the suggested strategy lists have statements like "convince yourself that you can learn enough math" (Proga, p. 11) , to indicate that emotions have to recognized in developmental mathematics courses.

Traditional Study Techniques

Traditional topics in study techniques comprise a large portion of mathematical study skills texts. Work habits are covered in every text reviewed, and issues relating to problem solving are given attention in most of the texts. Test preparation and test-taking techniques are also common to most of the texts. On the other hand, time management, how to read the textbook, previewing before class, and notetaking are subjects that are often lightly covered or not covered at all. The different levels of detail between general study techniques texts and these texts is somewhat unexpected, but as Nolting notes, "Math courses are not like other college courses. Because they are different, they require different study procedures." (Nolting, p. 21)

Work habits are generally considered important for success. Richard Smith suggests that students "be overdedicated for the first two to three weeks of the course" in order to get off to a good start. (Smith, p. 24) Arem warns students about the need to stay current in class and to not fall behind (Arem, p. 102), while Lorraine Gregory devotes several pages to the idea of "getting the work done, even when you would rather not". (Gregory, pp. 5 - 7) All mention the need to work consistently in math and the dangers of falling behind.

It is evident that George Polya's sixty year old problem solving techniques still dominate, for half the texts reviewed base their problem solving sections directly on his work. However, elaborations are common. Arem expands Polya's four steps into a twelve step approach (Arem, pp. 162, 163), while Ooten adds the ideas of "mind-shifting" and fifteen substeps. (Ooten, pp. 176 - 191) Gregory takes Polya's steps verbatim, but attempts to show their interrelation with metacognition. (Gregory, p. 19) In all cases, problem solving is made out to be more artistic than formulaic, with a "bag of tricks" and "no one best way" to solve problems. (Ooten, p. 176)

Test preparation is considered a highly important skill in the majority of the texts reviewed. It is generally accepted that "it is never too early to begin studying for a math test". (Proga, p. 22) The study process begins by making a list of all the possible topics that may appear on the test. (Smith, p. 99) One then commits to "overlearning" the topic. (Arem, p. 112). However, Smith notes that "overlearning" is in reality just learning, and he suggests working on a single topic at a time. According to Smith, "total confidence ... requires doing more problems than you would do if you merely desired to achieve a general familiarity with the topic." (Smith, p. 118) Nolting further cautions the student to not simply work homework style problems, but to make practice tests. Time constraints and anxiety exist in a testing situation, which is considerably different from the open book, open note, repeatedly similar problem format common to homework sets. (Nolting, p. 184)

Once the studying has completed, all the authors recommend a good night's sleep. However, some authors go much further with their advice. Arem suggests that students dine on low-fat protein foods like fish, skinless chicken, and tofu, and advises that students "avoid large meals and high-fat foods before tests because they tend to dull the mind and slow mental process." (Arem, p. 147) Ooten also recommends protein, and notes that "carbohydrates may make you tired". (Ooten, p. 202) Whether it is helpful to arrive early (Smith, p. 141) or just on time (Gregory, p. 27) is somewhat disagreed upon, but Ooten notes that one can absorb other people's anxiety, so it is best to ignore others before a test. (Ooten, p. 202).

The test-taking strategies proposed all take the same basic plan, but some authors are much more detailed than others. Nolting recommends that as soon as one receives a test, he should "turn it over and write down the information that you put on your mental cheat sheet." Then, after previewing the test, he recommends doing "a second memory data dump." (Nolting, p. 185) Other authors recommend a single "dump" or omit the idea entirely. However, all authors recommend previewing (also called skimming) the test, and many recommend doing the easier problems first. (Nolting, p. 186; Smith, p. 142; Gregory, p. 27) The advice differs in the amount of detail and to some extent on topic after that, with between nine and nineteen total steps listed. These include "...focus on remaining calm, relaxed, and positive" (Arem, p. 133), "show all your work and attempt to solve every problem" (Proga, p. 24), and "check all your work" (Gregory, p. 28).

Although most authors mention that the student should "use time wisely" (Ooten, p. 203) or "don't be in a rush to leave the room" (Smith, p. 144), only Nolting actually addresses the issue of leaving the room early. He writes, "Even though we encourage students to work until the end of the test period most students leave the classroom before the end of the period." The first reason for this is because "...test anxiety gets so overwhelming that ... the relief from the test anxiety ... is worth more than getting a better grade." The other reason is that students perceive that "students who turn their tests in last are 'dumb and stupid' " (Nolting, p. 187) These highly emotional responses to testing suggest that the affective aspect is found even within traditional study techniques.

The majority of authors discuss time management issues, and for the most part the information is unsurprising. Creating a schedule of your time utilizing broad categories (sleep, work, class, etc.) is typical of the reviewed texts. A few creative additons must be noted however. Nolting suggests having a separate "weekly study-goal sheet", which takes the time a student has allocated for studying and allocates it to specific classes. According to the author, this process can help you "establish the best time to study math." (Nolting, pp. 111, 112) Arem also suggests the students subdivide their studying, but in her case she is advising that students establish a personal reward system. If a student determines that he has worked effectively, then Arem recommends that the award himself with a "point" for every fifteen minutes of effective effort. These points can then be self-redeemed for rewards like taking time off to see a movie or for a snack break.

Of the authors that mention the textbook, all note that "to use a mathbook effectively, you cannot simply 'read' it in the same way you would read a novel or a history book." (Proga, p. 20). The idea of "previewing", or reading the text before class even if it does not make complete sense, is a typical suggestion. (Ooten, p. 160; Gregory, p. 9) Arem provides the more detailed suggestion that "math reading assignments need to be tackled at least three times," (Arem, pp. 120 - 122) which is to say that a survey of main points and highlighted areas needs to be accomplished before class, an in-depth reading needs to take place soon after class, and a third reading for review needs to take place prior to the test. Other advice given is mostly what one would expect (highlight or underline important points, pay particular attention to tables and other illustrations, etc.), but one point several texts made was to pay particular attention to definitions. Gregory notes that "definitions are an important part of the content in mathematics texts, (Gregory, p. 9), and Nolting adds that although instructors of non-math courses may suggest that their students skim over unknown words in order to try to pick up the meanings from the context, in mathematics "skipping some major concept words ... [means] you will not understand what you are reading and therefore will not be able to do the homework". (Nolting, pp. 158, 159) Vocabulary is important.

One can get a fairly complete view of math study techniques from reading multiple texts, but there really isn't a single source I would recommend. Even with as important a topic as notetaking, one can find anything from a detailed three column approach (Nolting, pp. 144 - 150) to a few hundred word summary of tips and suggestions. (Arem, pp. 103, 104) Still, considering the idea that mathematics students could benefit from specialized study techniques didn't really exist until a couple of decades ago, I'd say that progress is being made.

Citations:

Arem, C., Conquering Math Anxiety, 2nd ed., Pacific Grove, CA: Brooks/Cole Publishing Company, 2003

Gregory, L., Studying Mathematics: A Handbook for Reluctant Mathematics Students, Dubuque, IA: Kendall/Hunt Publishing Company, 1999

Nolting, P., Winning at Math: Your Guide to Learning Mathematics Through Successful Study Skills, 4th ed., Bradenton, FL: Academic Success Press, 2002

Ooten, C., Managing the Mean Math Blues, Upper Saddle River, NJ: Prentice Hall, 2003

Proga, R., Math for the Anxious: Building Basic Skills, New York: McGraw-Hill, 2005

Smith, R. M., Mastering Mathematics: How to Be a Great Math Student, 3rd ed., Pacific Grove, CA: Brooks/Cole Publishing Company, 1997

Math Study Skill Text Reviews:

Mastering Mathematics – How To Be A Great Math Student, third ed., by R. M. Smith. (copyright 1997)

Mastering Mathematics begins with a survey whose questions closely align with the text's main topics: attitude control, work habits, and test preparation techniques. To introduce the first of these, the author states that "low ability" can be overcome by attention to detail and persistence. He then provides specific techniques for overcoming a weak background and suggests that one should not make excuses and that one should take a positive approach towards the course, even if positive feelings don't actually exist. This encouragement to persist at all costs and to force oneself to be positive is more specific and in my opinion more applicable to the generic "be positive" techniques used by other texts.

The author's treatment of work habits arrange themselves around four "key steps": following the teacher's explanations in class, mastering the material, doing work in a timely manner, and following an organized test study plan. Each of these steps is assigned its own short chapter, but the information provided is generic. For example, students are told to not miss class, to take notes, and to ask questions. I found the author's mention of the two column (Cornell) method to be quite clear, and I liked his discussion of note cards. Unfortunately, some of the examples utilize algebra, which might increase the anxiety of the LERN student.

Five of the thirteen chapters in the book can be categorized as dealing with test preparation, but the author avoids any mention of test anxiety. Instead, he refers to these emotions as a "mental block" and explicitly states that these mental blocks are the result of poor study habits. It is hence inferred that if the student always aims for near 100%, begins to prepare for the test well in advance, and masters each topic one topic at a time, mental blocks will fail to exist. While I strongly doubt this is the case, his advice on test preparation is generally sound, and his repeated insistence on mastering one topic at a time is not something I have seen elsewhere.

Positives:

- practical advice on how to set a positive attitude
- good treatment of how to use flash cards and notes
- clear advice on test preparation techniques

Negatives:

- moderately high reading level
- examples often use algebra
- unusual treatment of anxiety issues

Managing the Mean Math Blues, by C. Ooten (copyright 2003)

The term "blues" hints at both a form of depression and a form of music, and it therefore comes as no surprise that the author claims expertise as both a psychotherapist and a jazz pianist. The tone of the book borders on the "touchy-feely", but the information presented is of good quality nonetheless. The book opens with a section on "challenging negative beliefs" and then over the next few chapters the text describes in detail how behaviors and emotions are interlinked, the types of distortions people make in their thinking, and intervention strategies for negative thoughts. While I found these chapters very interesting, the wording often surprised me. For example, I don't think I have ever had the useful thought "Situations like this are exactly what I need to push my boundaries out and move into new territory." While it isn't difficult to figure out the author's intent, I think that these curious phrases made it more difficult to connect the author's points to my life as a scholar.

Part three of the book deals with a discussion of Gardner's multiple intelligences hypothesis and of learning modalities. Although these chapters are rather predictable, it is nice to note that most of the examples presented involve fractions, an appropriate topic for the LERN curriculum. The final part of the book (involving fully half the page count) presents a variety of useful problem solving and study strategies. I found the chapters on webbing (also called mind-mapping) and problem solving to be particularly well written, and the fifteen short tips in chapter 15 (Skills to Bridge the Gaps) provide an excellent brief introduction to the art of being a college student. Although the author notes in chapter two that the chapters do not have to be read in order, enough references to part two are made in part four for me to suggest that the text be read in its traditional order.

Positives:

- good treatment of the interactivity of behaviors, thoughts, and emotion
- clear instructions on webbing and problem solving
- strong emphasis on the affective issues related to mathematics
- examples usually use fractions or other low-level topics

Negatives:

- occasionally artificial "touchy-feely" wording disrupts the intended message
- strong emphasis on understanding ones feelings could turn off students

Conquering Math Anxiety, by C. Arem (copyright 2003)

Conquering Math Anxiety appears intended for the serious mathphobe. Although the exercises and activities would benefit most students, the tone of the text suggests that the author (a counselor) deals mainly with a subset of students. One student mentioned early in the text panics, runs out of the room, and vomits uncontrollably. Another has hated math since she lost control of her bladder while working a problem at the chalkboard at the front of the class. These examples might serve to show students how severe anxiety can be, but I also suspect that the less anxious might wonder why they are reading the book.

The book begins with a narrative of some of the causes of math anxiety. Included in these early chapters are exercises such as writing a math autobiography and describing one's mathematical goals. The short chapter three begins to describe a technique for managing anxiety through meditation and deep breathing exercises. These exercises are also included on an included audio CD, which steps the listener through two different relaxation exercises while sounds of birds, water, and electronic synthesizers play in the background. I found the CD to be a very well done variation of a time-tested method of anxiety control which I believe would be beneficial to many students if they can be convinced to use it.

Chapters five and six note the importance of a positive attitude and the value of changing negative or disempowering statements to more useful positive statements. This topic is continued on the CD on two much longer tracks (10 and 26 minutes). Tracks similar to these have been around for a while and have been shown to be effective with many students. Yet again, a certain amount of suspicion must be overcome. One of the exercises in the text suggests that after visualizing a period in your life where you felt confident, you should snap your fingers. According to the author, if this is done often enough, one can then create a feeling of confidence just by snapping your fingers. While this may be so, I find myself wondering how many students would do this activity frequently enough to make the necessary mental connection.

The middle of the text includes four chapters that deal with math learning styles, math study skills, studying for tests, controlling test-based anxiety, and problem solving. These four chapters (seven through ten) provide solid advice without the extreme examples mentioned earlier in the text, and in my opinion would serve as a good study techniques text taken on their own.

The learning styles chapter is particularly interesting, for the the text goes beyond mere modalities (visual, audial, haptic) and asks the student to consider their preferred learning time, background sound level, lighting level, temperature, and food level, among other factors. The study skills chapter offers the typical set of tips, but also has the unusual (though retrospectively obvious) suggestion of making a reward chart as a means of motivating oneself. The testing chapter glosses over how to prepare academically for a test (using only six pages), but goes into significant detail on how to deal emotionally and physiologically for a test. This includes having a summary of the main points of chapters five and six and a discussion of the effects of diet on the mental process. Finally, the problem solving chapter mentions Polya's work, but it does not make this classic technique the focus of the chapter. Instead, the author mentions her own twelve-step technique and suggests playing games like Chess or Clue to strengthen one's logical sequential thinking skills. I rather like the author's twelve-step approach, but I don't believe that students could memorize it well enough for it to have value during a test.

Positives:

- included audio CD provides guided meditation exercises
- detailed treatment of learning styles and the emotional aspects of learning
  - practical advice on how to set a positive attitude

Negatives:

- extremely negative examples in early chapters could turn off students
- many exercises need frequent repeating to provide a benefit

<u>Studying Mathematics -- A Handbook for Reluctant Mathematics Students,</u> by L. Gregory (copyright 1999)

One of the thinnest handbooks reviewed (at only 60 pages), Studying Mathematics should be one of the least expensive handbooks to purchase, and its lack of verboseness might not be that great a handicap. The text begins with suggestions on "getting the work done, even when you would rather not", and uses a format of annotated "to-do" lists. Thus, one item on the list might be to "review lecture notes", and then the text uses a paragraph or so to clarify just what is implied by this list item. This formal allows for considerable conciseness, but the lack of detail means that some detail would have to be filled in by the instructor. Clearly, this is not a self-help book.

The subjects included are what to do after class (and when to do it), how to read a math textbook, how to take notes, how to study for a test, and problem solving. At the end is a single page entitled “Secrets for Success”, which gives a brief potpourri of tips like “find a study friend” and “attend every class”. Notably absent are issues relating to anxiety -- the entire topic of anxiety management is given less than half a page. Considering all these techniques combined cover just 30 pages, the quality of discussion is quite good, and gives me the feeling that the text probably derived from a series of 2 - 3 page handouts some years ago.

The remainder of the handbook is composed primarily of eight worksheets that relate to the points given in the text. Most of these are just two pages in length, and it would likely take only 15 - 20 minutes to complete each one. Hence, I estimate that the entire workbook could be covered in 4 - 5 hours, which is probably about the maximum time that would be available considering the LERN curriculum. Unfortunately, most of the examples are taken from pre-algebra, and as such would be difficult for LERN students.

Positives:

- brevity is a blessing considering the packed LERN curriculum
- moderately low reading level
- provided lists give context to the suggested activities

Negatives:

- extremely brief treatment requires instructors to clarify material
- inadequate discussion of anxiety
- many examples beyond the level of the LERN curriculum

Winning at Math, by P. D. Nolting, fourth edition (copyright 2002)

Winning at Math and its accompanying workbook “Math Study Skills”, form a detailed treatment of mathematics study skills, and as such would be suitable for a semester long course in the subject. The first part of the text deals with how mathematics is different from other subjects and hence how the skills needed to succeed in math are different from other subjects. The author states that a full quarter of a student’s success is due to affective characteristics (motivation, anxiety, study habits) and that another full quarter of a student's success is due to the effectiveness of the instructor, but he provides no references for this. While I fully agree that affective characteristics and instructor effectiveness are a considerable portion of a student's success, I dislike seeing opinions enumerated in such a way. The author goes into quite a bit of detail regarding learning modalities and styles, how human memory works, and memory techniques like mnemonic devices and acronyms. Unfortunately, he also suggests the student take five outside assessments, noting in the text only that one should "consult your instructor ... for assistance in taking these assessments." Clearly, this is not a self-help book.

Beginning with chapter four, the advice becomes more specific and on a stated topic. Chapter four discusses math anxiety, from its definition to its control. Although the author considers math anxiety to actually have three fairly independent components, all three components have similar causes and means to control, for according to the text, anxiety is a learned behavior that can be unlearned. The reason anxiety affects learning and testing is nicely described via the model of human memory given in chapter three, and the chapter ends by not only giving physical relaxation techniques but by also warning of the dangers of negative self-talk.

Since the author considers procrastination to be a severe form of math avoidance behavior, the next chapter promotes a specific format for planning one's schedule and for creating a weekly study sheet. Advice is provided as to how to modify the schedule as time passes, the dangers of being overcommitted, and the importance of keeping one's studying to short spurts of thirty to forty-five minutes. I found this chapter to be quite well done, except that the suggested format for scheduling oneself strikes me as being overly detailed.

Chapter six deals with the importance of creating a positive study environment, and describes desired traits of the environment one should have when studying alone, with a group, or with a tutor. It is assumed that the student will have access to a tutor and to a math lab and will make use of both, which certainly isn't true of all students everywhere, but is true for LERN students at Mt. SAC. I particularly liked the author's advice of collaborative groups, for the author not only described the tasks of a good group member, but also made suggestions for handling a poor or disruptive group member.

The seventh chapter describes how to improve one's note taking skills. Quite a bit of the information in this chapter is commonly described elsewhere, but once again the author has particular preferences that are stated as fact. For example, a "golden triangle" for seating is mentioned that includes the front row and the center back seat of the classroom. Although one often sees advice to sit towards the front of a classroom, this triangle is not commonly accepted as being superior seating. Still, the author helpfully describes in detail the three column note taking system, how and when to rework or review your notes, and how to use note cards, and I liked how the author gave importance to the study of vocabulary.

As a LERN instructor, I have frequently found that many students fail to read their textbook, and so it is gratifying to discover that this book devotes a chapter to reading and homework techniques. The reading of a math text is explained using a detailed nine-step approach that makes it clear that reading mathematics involves much more than reading. The detail provided also makes it clear again that this isn't a book to be left to the student. In my opinion, students would only adopt such detailed and time consuming reading skills if they were reinforced by an instructor. A similarly detailed ten-step process for doing homework and some examples finish the chapter. Some of the material in the examples is within the realm of the LERN curriculum, but some algebra exists as well.

One of the best written chapters in the text comes next and deals with how to improve test taking skills. The author explains how to take a test, why simply attending class and doing practice homework are not sufficient, and how to avoid common errors. He also describes the importance of analyzing one's test after it is returned. Surprisingly, he does not review the techniques from the fourth chapter relating to test taking anxiety, but this is a small omission easily corrected by the instructor.

Aside from two chapters dealing with the special cases of the distance-learning student and the learning disabled student, the final chapter is entitled "How to Take Control of Math", and it singles out for discussion the ideas of self-worth and self-esteem. According to the author, the action of taking responsibility for one's success is predicated on having an internal locus of control; that is, a belief that one controls ones own actions, and that poor instructors, home conditions, finances, etc. are simply challenges to be faced as opposed to reasons to expect to fail. Therefore, the author discusses how to avoid feelings of helplessness, how to overcome procrastination, and how to improve self-esteem. I would have preferred this chapter towards the beginning of the text, but the suggestions provided are solid, and constitute a point from which to begin a class discussion or individual counseling session.

Positives:

- most detailed and in depth of the texts reviewed
- companion workbook provides support to the topics being covered

Negatives:

- material is sufficient for a complete stand-alone study techniques course
- occasionally states personal opinions or views as established fact
- considerable reading and follow-through required of the student

Math for the Anxious: Building Basic Skills, by R. Proga (copyright 2005)

Math for the Anxious begins with a brief discussion of why and how we learn math and why math anxiety is something that needs to be addressed. Then comes a chapter that briefly covers such topics as attitude, strategies for effective studying, test preparation, and test-taking. This effectively ends the part on anxiety and study techniques.

What follows in the next eight chapters is a brief review of arithmetic topics, with one each devoted to whole numbers, fractions, decimals, percents, graphs, signed numbers, measurement, and geometry. Each of these topics begins with a few negative "math memories" such as "All I remember is that two negatives make a positive, but I have no idea what that means."

According to the preface, the purpose of such an introduction to each chapter is "so that readers will realize they have are not alone and that their challenges in math are surmountable." Perhaps this is so, but I found that by chapter eight this constant negativity was rather depressing. Then a couple of paragraphs are used to alert the reader to common problems and to provide hints for studying the topic. Finally, the bulk of the chapter is an abbreviated discussion of the mathematical topic. I found the procedures to be well explained and the graphics to be detailed and well placed. However, I also found the writing to be rather dense. Although most college students would not find the reading level difficult, there's a lot to understand on each page, and there is no use of color or arrows to help clarify the examples. Still, the author does a decent job of presenting the material, and each chapter ends with helpful strategies to help one learn the chapter's material.

Positives:

- brief and to the point
- functions as a second reference textbook for many LERN topics

Negatives:

- extremely brief treatment of each topic
- lack of color and annotation in examples
- negative "math memories" open each chapter

	Mastering Mathematics	**Managing the Mean Math Blues**	**Conquering Math Anxiety**	**Studying Mathematics**	**Winning at Math**	**Math for the Anxious**
Anxiety issues	Not covered -- low ability and mental blocks can be overcome	Myths are dispelled and shyness discussed	Myths, history, internal barriers, and other pressures discussed	Not covered	Myths, history, causes, and how anxiety affects learning are discussed	Causes and myths covered
Anxiety control	Not covered	Physical relaxation techniques covered	Visualization and physical relaxation techniques discussed in detail	Two relaxation techniques suggested briefly	Two relaxation techniques discussed; reader is directed to other text	Not covered
Attitude control/ Avoiding negativity	Must keep positive, persist, and attempt to do much better than just pass	Self-intervention strategies are covered	Chapter devoted to positive thinking techniques	Not covered	Brief coverage on avoiding negativity	Brief coverage
Work habits/time management	Work habits are covered in depth, but time management is only lightly mentioned	Work habits are covered in depth, but time management is only lightly mentioned	Both are lightly covered -- notes that mental energy varies with time of day	Work habits are lightly covered, but time management is not mentioned	Both covered in detail (two chapters)	Work habits are lightly covered, but time management is not mentioned
Reading the textbook/ preparing prior to class	Doing homework soon is important, but reading techniques are not covered	Previewing is mentioned briefly	Both are lightly covered	Both are lightly covered	Reading techniques are covered, but preparing prior to class mentioned only in passing	How to read a math book is mentioned briefly
Following the instructor/ note-taking	Complete notes are suggested, but no techniques are suggested	Not covered	Both are lightly covered	Both are lightly covered	Covered in detail, although unusual notetaking and seating methods suggested	Both are lightly covered
Webbing/ mind mapping	Not covered	Explained in detail with specific suggestions for math	Not covered	Not covered	Not covered	Not covered

Matrix of How Each Text Deals With Study Skills and Anxiety Issues

	Mastering Mathematics	**Managing the Mean Math Blues**	**Conquering Math Anxiety**	**Studying Mathematics**	**Winning at Math**	**Math for the Anxious**
Test-preparation/ memory techniques	Extensive coverage (five chapters)	Both covered in detail (two chapters)	Part of the study skills chapter -- advice is good but brief	Briefly covered	Covered in appendix F -- very brief	Test preparation is briefly covered, but no specific techniques mentioned
Test-taking	Covered, including time and post-analysis issues	Ten short tips mentioned over three pages (202 - 204)	Covered, including calming, health, and diet issues	Briefly covered, including post-analysis issues	Covered in detail, including post-analysis issues and common error types	Briefly covered, including post-analysis issues
Cooperative and collaborative learning issues	Not covered	Not covered	"Study-buddies" mentioned briefly (pages 125 & 126)	Not covered, except "find a friend" is a suggested learning technique	Benefits and characteristics covered as is how to handle problems	Not covered, except that joining a study group is mentioned as a learning technique
Value of the affective domain	Not valued	Valued -- negative feelings towards math and teachers a serious reason for failure	Valued -- emotional anxiety listed as a common reason for failure, though not the only one	Not valued	Highly valued -- author states that affective issues account for 25% of typical grade	Valued -- frustration embarassment, and poor interactions listed as common reasons for failure
Problem solving	Not covered	Polya's technique is mentioned and expanded upon	Polya's technique is mentioned and expanded upon with a twelve point system	Polya's technique is interwoven with meta-mathematical ideas	Mentioned in several places in the text, but an unusual ten step approach suggested	Mentioned as important, but no techniques given
Learning styles	Not covered	Modalities (Visual, audial, haptic) covered	Modalities covered along with temporal, environment, and food preferences	Not covered	Modalities covered along with "cognitive learning styles" and environment	Modalities covered briefly; existance of other factors mentioned
Intelligences	Not covered	Gardner's multiple intelligences model discussed	Not covered	Not covered	Not covered	Not covered

Matrix of How Each Text Deals With Study Skills and Anxiety Issues

Phase III – Scope and Sequence of Math Texts

Unlike the previous two phases of my sabbatical, this phase does not have a formal report. Instead three tasks were undertaken. The first was to identify which of the selected textbooks approached mathematical topics in a non-traditional manner. None did, although the book by Tussy and Gustafson did have more higher order questions in the problem sets.

The second and third tasks were to compare the arithmetic texts' scope and sequence to each other and to compare the objectives of the textbook that LERN math courses used in 2007 with the objectives of the textbook that the Math pre-algebra course used in 2007. This information is presented in tabular form on the pages that follow.

Notes on the table:

1) If the objective is addressed in multiple sections, only the first two or three sections are listed.
2) The "Bittinger 1" text is intentionally missing chapters 7, 8, and 9, so the objectives' subsections are unknown. These objectives are marked with an asterisk (*).
3) Objectives that deal with signed numbers, and not only integers, are marked with a plus (+)

Abbreviation Key:

Bittinger 1 – Basic Mathematics, 10th ed., by M. Bittinger (2007) [the current LERN 48/49 textbook]

Carson – Prealgebra, 2nd ed., by T. Carson (2005) [the current MATH 50 textbook]

Bello – Basic College Mathematics, 2nd ed. by I. Bello (2006)

Bittinger 2 – Basic Mathematics with Early Integers, by M. Bittinger and J. Penna (2007) [nearly identical to Bittinger 1 save for the treatment of integers]

Staszkow – Math Skills, 6th ed., by R. Staszkow (2003)

Tussy – Basic Mathematics for College Students, by A. Tussy and R. Gustafson (2007)

Van Dyke – Fundamentals of Mathematics, 9th ed., by J. Van Dyke, J. Rogers, and H. Adams (2007)

	A	B	C	D	E	F	G	H	I	J
1				Scope and Sequence for Selected Textbooks						
2	Objective	Bittinger 1		Carson		Bello	Bittinger 2	Staszkow	Tussy	Van Dyke
3										
4	Meaning of digits in standard not.	1.1a		1.1.1			1.1a	1.1	1.1	1.1.1
5	Standard to expanded notation	1.1b		1.1.2		1.1ab	1.1b	1.9	1.1	
6	Standard <-> word name	1.1c		1.1.3		1.1cd	1.1c	1.1		1.1.1
7	Adding whole numbers	1.2a		1.2.1		1.3a	1.2a	1.3, 1.4	1.2	1.2.1
8	Perimeter with whole numbers	1.2b		1.2.3		1.3b	1.2b		1.2	1.2.4
9	Adding and subtracting sentences	1.3a		1.2.5			1.3a		1.3	
10	Subtracting whole numbers	1.3b		1.2.4		1.4a	1.3b	2.1, 2.2	1.3	1.2.2
11	Rounding to nearest given place	1.4a		1.1.5		1.2b	1.4a	1.2	1.1	1.1.3
12	Estimating +/- (whole numbers)	1.4b		1.2.2			1.4b		1.2, 2.6	1.2.3
13	Use < and > (whole numbers)	1.4c		1.1.4		1.2a	1.4c	1.9	1.1	1.1.2
14	Multiplying whole numbers	1.5a		1.3.1		1.5a	1.5a	1.5, 1.6, 1.7	1.4	1.3.1
15	Multiplying by a power of 10					1.5b		1.8	1.4	1.5.2
16	Estimating x (whole numbers)	1.5b					1.5b			1.3.2
17	Area with whole numbers	1.5c		1.3.5		1.5c	1.5c		1.4	1.3.3
18	Mult. and div. sentences	1.6a		1.4.2		1.6a	1.6a			
19	Dividing whole numbers	1.6b		1.4.1		1.6b	1.6b	2.3 through 2.7	1.5	1.4.1
20	Equations via trial and error	1.7a					1.7a	7.7		
21	One step equations	1.7b		1.2.5, 1.4.2		1.9ab	1.7b	7.8 through 7.12	8.1, 8.2	1.4.end
22	Problem solving	1.8a		1.7.1		1.9c	1.8a	7.14		
23	Exponential notation	1.9ab		1.3.3&4, 2.4.2		1.7c	1.9ab	1.9, 7.15	1.6	1.5.1
24	Order of operations (whole num.)	1.9c		1.5.1		1.8a	1.9c	2.8	1.7	1.6.1
25	Nested parentheses	1.9d		1.5.1		1.8b	1.9d		1.7	1.6.1, 3.10.1
26	Finding factors of a number	2.1a		3.6.3		1.7b	3.1a	2.9	1.6	2.3.1&2
27	Multiples	2.1b					3.1b		3.5	2.2.1&2
28	Finding if a number is divisible	2.1b					3.1b	2.9	1.5	
29	Prime vs. composite	2.1c		3.6.1		1.7a	3.1c		1.6	2.4.1
30	Prime factorization	2.1d		3.6.2		1.7bcd	3.1d	2.9	1.6	2.5.1&2
31	Div. by 2, 3, 5, etc.	2.2a					3.2a	2.9	1.5	2.1.1&2
32	Numerator/Denominator	2.3a					3.3a	3.1	3.1	3.1.1
33	Write fraction notation for a set	2.3a		5.1.1		2.1a	3.3a	3.1	3.1	3.1.1
34	Special fractions (n/n, 0/0, etc.)	2.3b		5.1.3		2.1b	3.3b		2.5	
35	Proper vs. Improper fractions					2.1bcd		3.1	3.1	3.1.2
36	Whole number x fraction	2.4a				2.3a	3.4a	3.4	3.2	3.3.1
37										

	A	B	C	D	E	F	G	H	I	J
38				Scope and Sequence for Selected Textbooks						
39	**Objective**	Bittinger 1		Carson		Bello	Bittinger 2	Staszkow	Tussy	Van Dyke
40										
41	Multiplying fractions	2.4b, 2.6a		5.3.1&2		2.3a	3.4b, 3.6a	3.4, 3.7	3.2	3.3.1
42	Equivalent fractions (x n/n)	2.5a		5.1.4, 5.5.4		2.2a	3.5a	3.2	3.1	3.5.1&2
43	Simplifying fractions	2.5b		5.2.1&2		2.2b	3.5b	3.2	3.1	3.2.1
44	Test fractions for equality	2.5c					3.5c		6.2	
45	Reciprocals	2.7a		5.4.1		2.3d	3.7a	3.5	3.3	3.3.2
46	Dividing fractions	2.7b		5.4.1		2.3d	3.7b	3.5	3.3	3.3.3
47	Equations with fractions	2.7c, 3.3c		5.4.5		2.7abc	3.7c, 4.3c		8.1, 8.2	3.4.end
48	Least common multiple (LCM)	3.1a		5.5.1&2		2.4c	4.1a	'3.10	3.4	2.6.1&2
49	Greatest Common Factor (GCF)			3.6.4&5		2.2b			3.4	
50	Adding fractions	3.2a		5.6.1, 5.6.3		2.4abde	4.2a	3.8, 3.9, 3.10	3.4	3.6.1&2
51	Subtracting fractions	3.3a		5.6.1, 5.6.3		2.4e	4.3a	3.8, 3.9, 3.10	3.4	3.8.1
52	Use < and > (fractions)	3.3b		5.1.5		2.2c	4.3b	3.13	3.4	3.5.3
53	Fractions <-> Mixed numbers	3.4a		5.1.6&7		2.1cd	4.4a	3.3	3.5	3.1.3&4
54	Quotients as mixed numbers	3.4b					4.4b			
55	Adding mixed numbers	3.5a		5.6.5		2.5abd	4.5a	3.11	3.6	3.7.1
56	Subtracting mixed numbers	3.5b		5.6.6		2.5cd	4.5b	3.11	3.6	3.9.1
57	Multiplying mixed numbers	3.6a		5.3.3		2.3b	4.6a	3.6	3.5	3.4.1
58	Dividing mixed numbers	3.6b		5.4.2		2.3e	4.6b	3.6	3.5	3.4.2
59	Order of operations with fractions	3.7a		5.7.1		2.6ab	4.7a	3.12	3.7	3.10.1
60	Estimating fractions	3.7b					4.7b			
61	Decimal word names	4.1a		6.1.2		3.1a	5.1a	4.1	4.1	4.1.1
62	Fractions <-> Decimals	4.1b, 4.5a		6.1.1, 6.4.2		3.3abc	5.1b, 5.5a	4.9	4.1, 4.5	4.2.1, 4.7.1
63	Use < and > (decimals)	4.1c		6.1.4		3.4a	5.1c	'4.10	4.1	4.2.2
64	Rounding to nearest given place	4.1d		6.1.5		3.2d	5.1d	4.2	4.1	4.1.2
65	Adding decimals	4.2a		6.2.1		3.1c	5.2a	4.3	4.2	4.3.1
66	Subtracting decimals	4.2b		6.2.2		3.1d	5.2b	4.4	4.2	4.3.2
67	Equations with decimals	4.2c, 4.4b		6.2.7		3.5ab	5.2d, 5.4b		8.1, 8.2	4.3.end
68	Multiplying decimals	4.3a		6.3.1		3.2a	5.3a	4.5	4.3	4.4.1
69	Mult. dec. w/a power of 10					3.2b				4.5.1
70	4.7 million < - > 4,700,000	4.3b					5.3b			
71	Dividing decimals	4.4a		6.4.1		3.2c	5.4a	4.6, 4.7, 4.8	4.4	4.6.1
72	Order of operations with decimals	4.4c		6.5.1		3.4c	5.4c	4.11	4.3, 4.4	4.8.1
73	Repeating decimals	4.5b		6.4.2		3.3c	5.5b		4.5	4.7.1
74										

	A	B	C	D	E	F	G	H	I	J
75				Scope and Sequence for Selected Textbooks						
76	**Objective**	**Bittinger 1**		**Carson**		**Bello**	**Bittinger 2**	**Staszkow**	**Tussy**	**Van Dyke**
77										
78	Fractions and decimals together	4.5c		6.5.2		3.4bc	5.5c	4.11	4.5	
79	Estimating decimals	4.6a					5.6a		4.4	4.8.2
80	Ratios as fractions	5.1a		7.1.1		4.1a	6.1a	5.1	6.1	5.1.1
81	Simplifying ratios	5.1b		7.1.1		4.1a	6.1b	5.1	6.1	5.1.1
82	Rates	5.2a		7.1.3&4		4.2a	6.2a		6.1	5.1.2
83	Unit rates	5.2b		7.1.5		4.2b	6.2b		6.1	5.1.3
84	Test ratios for proportionality	5.3a		7.2.1		4.2cd	6.3a	5.2	6.2	5.2.1
85	Proportions with missing value	5.3b		7.2.2		4.2b	6.3b	5.2, 5.3	6.2	5.2.2
86	Proportions of similar shapes	5.5ab		7.2.4			6.5ab			
87	Percents as fractions or decimals	6.1a				5.1a	7.1a	5.5	5.1	6.1.1
88	% -> fraction	6.2b		8.1.1		5.1c	7.2b	5.5	5.1	6.3.2, 6.4.1
89	% -> decimal	6.1b		8.1.2		5.1a	7.1b	5.5	5.1	6.2.2, 6.4.1
90	fraction -> %	6.2a		8.1.3		5.1d	7.2a	5.6	5.1	6.3.1, 6.4.1
91	decimal -> %	6.1b		8.1.4		5.1b	7.1b	5.6	5.1	6.2.1, 6.4.1
92	% problems -> percent equations	6.3a		8.2.1		5.2abc	7.3a	5.7	5.2	6.5.1
93	Percent problems ("What % of ..."	6.3b, 6.4b		8.2.1, 8.3.1		5.2abc	7.3b, 7.4b	5.7	5.2	6.5.1
94	% problems -> proportions	6.4a		8.4.1&2&3		5.3abc	7.4a	5.8		6.5.2
95	Measures of central tendency	7.1*		9.1.1&2&3&4		6.4abc	8.1abc	2.10, 6.8	1.7, 7.2	1.7.1&2&3
96	Perimeter of geometric figures	9.1*		1.6.1		8.1a	10.1a	8.6	9.5	7.3.1
97	Area of geometric figures	9.2*		1.6.2, 5.3.7		8.2abcd	10.2ab	8.6	9.5	7.4.1
98	Square roots	9.6*		1.4.4, 2.4.5		8.4a	10.6a	1.9	4.6	7.6.1
99	Integers as real-world situations	10.1a		2.1.1			2.1a		2.1	8.1.1+
100	Integers on the number line	10.1b		2.1.2		9.1a	2.1b	7.2	2.1	8.1.1+
101	Signed fractions -> decimals	10.1c		6.4.2		9.1a	5.5a		4.5	
102	Use < and > (Integers)	10.1d		2.1.3			2.1c	7.2	2.1	
103	Absolute value	10.1e		2.1.4		9.1b, 9.3b+	2.1d		2.1	8.1.2+
104	Adding integers	10.2a+		2.2.1&2&3		9.1cd, 9.3c+	2.2a, 5.2c	7.3	2.2, 4.2+	8.2.1+
105	Opposites (additive inverses)	10.2b		2.1.5		9.1a, 9.3a+	2.2b		2.1, 2.2	8.1.1+
106	Subtracting integers	10.3a+		2.3.1		9.1e, 9.3d+	2.3a, 5.2c	7.4	2.3, 4.2+	8.3.1+
107	Multiplying integers	10.4a+		2.4.1		9.2a, 9.3e+	2.4a	7.5	2.4, 4.3+	8.4.1&2+
108	Dividing integers	10.5a, 10.5c+		2.4.3		9.2a, 9.3f+	2.5a	7.5	2.5	8.5.1&2+
109	Recipocals of signed numbers	10.5b+		5.4.1		9.3f+	3.7a		3.3	8.5.1&2+
110	Order of operations with integers	10.5e+		2.5.1		9.4a+	2.5c	7.6	2.6	8.6.1+
111										

Phase IV – Supplemental Materials

The fourth phase of the sabbatical involved writing supplemental materials for the LERN 48 and 49 classes. Three issues were involved.

First of all, most arithmetic textbooks ignore integers until late in the semester, after fractions and decimals have been studied. Consequently, the texts contain fractional and decimal signed numbers in their integer problem sets. This added level of difficulty makes the study of integers in LERN 49 more difficult than it needed to be.

Secondly, both LERN 48 and 49 cover the topics of factors, prime numbers, fraction notation, and the multiplication of fractions. Since the same books are used for both courses, it was not uncommon for a LERN 49 student to spend the first two weeks doing precisely the same problems he or she had accomplished during the last month of LERN 48. This was frequently resented by the students as it was perceived as unnecessary work.

Thirdly, certain topics in the LERN 49 course are not always covered at an appropriate level by commercially available textbooks. LERN 49 is supposed to include the topics of perimeter, area, problem solving, and the arithmetic mean (average). For some reason, coverage of these topics by publishers is rather haphazard. Some books provide a level of detail with these topics that is entirely unnecessary, while others assume the issues are already known and the students can already work such problems before entering class.

As a solution to these issues, I wrote materials covering two units (18 objectives) for LERN 48 and two units (15 objectives) for LERN 49. The LERN 48 units assume that the student has already completed the portion of the course relating to study techniques and whole numbers. Consequently, the objectives involve factors, primes, and an introduction to fractions. Since adding and subtracting unlike fractions is not a topic covered in LERN 48, it was possible to place adding fractions before multiplying fractions, which allows multiplying to be treated as repeated addition. The LERN 49 units by contrast, are intended to be used at the start of the semester, replacing the first chapter or two of the adopted text. The first unit (8 objectives) deals with whole number topics, but not with the whole number operations, which are presumed to be known. However, the second unit (7 objectives) on integers for the most part utilizes the procedures for whole number operations, so the instructor should be able to identify which students still have difficulty with whole numbers at an early point in the class.

The layout that was used for the notes involves minimal verbiage, clear statements of new vocabulary, and procedures or "recipes" in bold text Where space permitted, annotated examples were provided. Every objective fits on a single page, without exception, and fully half the page is left blank save for a vertical bar. This space is intended for students to write two column (Cornell format) notes, should the instructor choose to not provide the notebook for the course.

Since my materials are intended to replace a commercial textbook, problem sets had to be created as well. Taking a cue from my research in phase I the worksheets were created with the following in mind:

1) The initial learning of a concept requires more than just a lecture. Students must have the opportunity to discuss and to reflect before a skill is mastered or the skill is likely to be mastered without regard to context.
2) A student is best engaged with a wide variety of problems even though such variety makes the assignment more difficult to complete.
3) Long-term knowledge of skills requires practice over a period of time.

Therefore, each worksheet starts with a moderately difficult problem or two. It is expected that some students will have to ask their study partners or the tutors for help, and that this brief re-teaching will help to solidify the concepts involved. Then come two or three higher order questions that ask the student to reflect or to explore a concept. Only then does the student find the more typical skills practice. However, only half the practice is given up front, and the remaining practice problems are provided over the next four worksheets.

Of course, much fine-tuning will be required, but I believe that regardless of the textbook the department may choose in the future, these materials will serve as a useful bridge between the LERN 48 and 49 curricula. The units are attached to this report as an appendix (on blue and green paper).

Phase V – Math Notebooks

The final phase of my sabbatical was spent creating notebooks for the LERN 48 and 49 classes. Part of the curriculum in both LERN mathematics courses involves the teaching of mathematical study techniques, and over the years I have attempted various ways of explaining the importance of time management and note-taking. However, people tend to continue their habits, and I found that once I stopped collecting and grading my students' notes, they almost always returned to their original preferences.

Insight for this phase arose some years ago when a number of deaf students remarked that they had considerable difficulty watching me, watching the ASL translator, and taking notes at the same time. They requested that I find someone whose notes could be copied for their use. But since few of my students were able to create high quality notes, I soon found using my classroom's SmartBoard to record and print the writings in the class.

Over time, I changed my approach and started preparing what would be projected on the board in advance. This considerably improved legibility while maintaining a record for my deaf students and the occasional other student who desired a copy after class. However, the rest of the class continued to transcribe my materials into their notes with little additional comment of their own. From the students' perspective, my materials <u>were</u> their notes.

To return the focus to the topic of note-taking, I have developed a format that I hope will improve my students' success. Although space is provided for time management, projects, and writing prompts, the bulk of the student notebook follows a two-page layout. On the left side I provide the recipes and definitions for every objective in the course. However, I provide few examples or illustrations. Below this are a handful of problems suitable for small-group or in-class work. The right side is mostly blank, with space provided for additional notes and for examples. It will be up to the instructor to provide both the additional information and guidance on how to improve ones notes, but with such substantial preorganization the end result should be that all information for an objective will be in a single place, but that the student will still have to analzye the material already preprinted before he can decide whether what is on the board should be copied. Although the most expermental aspect of my sabbatical (and unsupported as yet by any research) it is my hope that students will no longer consider notetaking to be equivalent to mindlessly copying. The notebooks' draft forms are attached to this report as an appendix.

Conclusion

My time on sabbatical has helped me gain a greater understanding of the current state of research in developmental mathematics in general and of teaching arithmetic to adult students in particular. I am grateful to have been given the time to sit, read, and reflect on how current trends should influence my teaching and the direction of my department. Although much of the recent research suffers from too many flaws to be considered conclusive, the new attention being given to my field by groups like NADE and AMATYC is reassuring, and I suspect that the conclusions of researchers will coalesce in the near future, for there is much agreement. In the near term, I expect to spend the next several years experimenting with the ideas to which I have been exposed.

The time spent reviewing textbooks and study skills books has also been educational, and the information I have gained should benefit my department as we select our next textbook. Our practice requires each professor to skim, assess, and rank each available text. All of the books I reviewed are still available in print, and as we begin the review process this spring I trust that the scope and sequence analysis in my sabbatical will be a helpful resource. Thus far, we have not required any of the commercially available study techniques texts in our courses, but I am hopeful that one will be adopted this year.

Perhaps the greatest benefit to the department will be the materials that I have created. There has been some talk of our department creating its own book, and although my work falls considerably short of that, it is reasonable to believe that we could eliminate outside materials for LERN 48 by next summer if we so desired. What I have done for part of the course could be duplicated for the rest given enough effort. Even if we retain professionally published materials, I expect my replacement units to serve as a widely adopted bridge between our two courses, and if they prove successful, that the notebooks will gain widespread use as well.

To my surprise, this sabbatical did not yield a great amount of material that could be adapted for conference presentation. Although I will likely develop a presentation, much of what we know about developmental education came about in the 1980s, when researchers like

Knowles and Gardner came up with new ways to think about learning and learners. Much of the past decade has apparently been spent adapting to these new paradigms, but signs exist that we will soon be able to point to conclusive large-scale research that should help to clarify the effective practices of our craft. Until then, the conclusions noted in my sabbatical will serve as guidelines for my advocacy of changes within my department's functioning as I continue seeking ways to improve the success of my students.

LERN 48

Notebook

Draft Edition

by Eric Kaljumägi

Table of Contents

A note to students:

Hello and welcome to LERN 48!

Part of any math course involves taking notes. But what exactly does that mean? Taking notes should certainly include writing down what the instructor places on the board, but it also includes capturing the most important material from your textbook. Although often overlooked, spoken instructions and tips from your fellow students are helpful too. Since this can be quite a bit to write, this notebook is intended to help you by already including the more common instructions and vocabulary terms. Without your input, this workbook is **not** a complete set of notes -- you will still have to stay awake and follow your instructor. However, by eliminating some of the more boring copying it is hoped that you will have more time to think about the material being presented.

A good notebook should also help you stay organized in other ways. Making good use of your time is an important skill, and this notebook provides space for you to keep your time organized. Vocabulary is an essential part of mathematics too, and so this notebook includes most of the words you will need to know.

Sad to say, the majority of the math classes you will take will not have such a notebook preprinted for you. However, there is no reason why you can't make a notebook like this one as you go along. It is hoped that by using this notebook, you will be encouraged to take better notes in the future. Good notes really help.

Please use the space below to note your instructor's instructions about this notebook. Does it have to be taken to class every day? Will you be turning it in for a grade? Do you need dividers or a math binder? Lets get off to a good start!

LERN 48 Time Management Project

How do you want to spend your time? Of course, you may spend your time as you please, but since you only have 168 hours to spend every week, it makes sense to spend your time wisely. Before beginning this project, think about how you intend to spend your time this semester.

A. Using the table on the next page, write down the categories of things you intend to do with your time this semester. ("Sleep", "class", "lab & schoolwork", and "relaxation" have been added for you, since I hope you intend some of each!)

B. On the schedule on the next pages, fill in the time slots that do not change times from week to week. These generally include your classes and your work schedule. Then draw boxes around these time slots in ink.

C. Fill in the times you intend to sleep. This should be 50 – 60 hours for the week, preferably at the same time each night. (Sleep is very important – it is very difficult to learn if you are tired.)

D. Add in the time you plan to do your labs and other schoolwork. You should have two hours of work time for each unit you are taking. (While this time doesn't have to stay fixed each week, most people do better by sticking to a schedule.)

E. Complete your schedule by listing the other things you must or want to do each week (housework, relaxation, etc.).

F. Using the table on the next page, add the hours for each category you selected. Make sure each day has 24 hours accounted for!

G. Answer the following questions:

Do I have at least 50 hours of sleep scheduled for my week? Y N
Is my "Lab & Schoolwork" time double my "Class" time? Y N
Do "Schoolwork & Lab", "In Class", and "Work" (if any)total less than 60 hours? Y N
Does my schedule have time for exercise and/or relaxation? Y N

IF YOU ANSWERED NO TO ANY OF THESE, REDO THIS SCHEDULE!
YOU HAVE A SCHEDULE THAT IS TOO DIFFICULT TO MAINTAIN!

Category	Time Spent	Time	Sunday	Monday	Tuesday
Sleep		Midnight			
Class		1 AM			
Lab & Schoolwork		2 AM			
Relaxation		3 AM			
		4 AM			
		5 AM			
		6 AM			
		7 AM			
		8 AM			
		9 AM			
		10 AM			
		11 AM			
		Noon			
		1 PM			
		2 PM			
		3 PM			
		4 PM			
		5 PM			
		6 PM			
		7 PM			
		8 PM			
		9 PM			
		10 PM			
		11 PM			

Time	Wednesday	Thursday	Friday	Saturday
Midnight				
1 AM				
2 AM				
3 AM				
4 AM				
5 AM				
6 AM				
7 AM				
8 AM				
9 AM				
10 AM				
11 AM				
Noon				
1 PM				
2 PM				
3 PM				
4 PM				
5 PM				
6 PM				
7 PM				
8 PM				
9 PM				
10 PM				
11 PM				

Appointment and Due Date Calendar

Use the calendars below to keep track of your assignments and appointments. Write the assignments you are given this semester (for all your classes) on the day they are due. You should also write the dates of tests, special events, and appointments here.

If you have a calendar system besides this one, ask your instructor if you may use your own system. Although it is a good idea to keep all your calendar information on a single calendar, your instructor may require that you turn in your calendar as part of your notebook!

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
___ Week 1	___	___	___	___	___	___
___ Week 2	___	___	___	___	___	___
___ Week 3	___	___	___	___	___	___
___ Week 4	___	___	___	___	___	___

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Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
— Week 5	—	—	—	—	—	—
— Week 6	—	—	—	—	—	—
— Week 7	—	—	—	—	—	—
— Week 8	—	—	—	—	—	—

Appointment and Due Date Calendar

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Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
— Week 9	—	—	—	—	—	—
— Week 10	—	—	—	—	—	—
— Week 11	—	—	—	—	—	—
— Week 12	—	—	—	—	—	—

Appointment and Due Date Calendar

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If you have a calendar system besides this one, ask your instructor if you may use your own system. Although it is a good idea to keep all your calendar information on a single calendar, your instructor may require that you turn in your calendar as part of your notebook!

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
— Week 13	—	—	—	—	—	—
— Week 14	—	—	—	—	—	—
— Week 15	—	—	—	—	—	—
— Week 16 -- Finals!	—	—	—	—	—	—

Objectives on this page: To find the place value of a number

Definitions:
digits – 0,1,2,3,4,5,6,7,8,9
periods – groups of three digits used in place value notation
Natural (counting) numbers – {1, 2, 3, 4, 5, …}
Whole numbers – the natural numbers and zero

Word spellings:
one, two, three, four, five, six, seven, eight, nine ten
eleven, twelve, thirteen, fourteen, fifteen, sixteen, seventeen, eighteen, **nineteen**
ten, twenty, thirty, **forty**, fifty, sixty, seventy, eighty, **ninety**
hundred, million, billion, trillion, quadrillion

Note: numbers between 21 and 99 use hyphens (ex.: 21 = twenty-one)

Note: "and" is not used in whole numbers
(ex.: 326 = three hundred twenty-six) <-- There's no "and"

Problems for class use:

In what place is the 5 in the number:

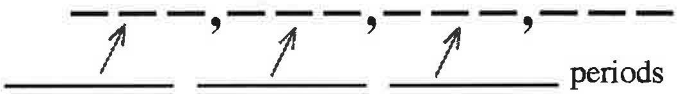
1) 752?

2) 5182?

3) 2,530,000?

4) 51, 243,604?

5) Fill in the blanks:



Date: ____________

Take extra notes here:

Place examples here:

Learning Tip:
Sit towards the front of the class or towards the middle of the room if you can. The back row tends to be more distracting since you can see everyone else, and the corners are usually harder to see and hear from.

Objectives on this page: Standard vs. expanded notation

Definitions: standard notation – a form of writing numbers where the value of each digit depends on its location in the number

expanded notation – a form of writing numbers where the place value of each digit is written out

To write a number in expanded notation:

1) Write down the first digit
2) Write out in words the place value for the first digit
3) If there is another digit, write a "+" and repeat these steps with the remaining digits.

Ex.: 51,000 in expanded form is 5 ten-thousands + 1 thousand

Note: Zeros need not be written out

Problems for class use:

Write in expanded form:

1) 752

2) 5,300

3) 2,530,000

Date: ____________

Take extra notes here:

Place examples here:

Learning Tip:
Reading the section that you will be covering before class gives you a head start on understanding the material. Don't worry if you don't understand everything -- any amount of previewing helps!

Objectives on this page: Word names vs. place value

When changing from place value form to word form, group the digits into periods of 3 (starting at the right) first. Then deal with each group separately.

Ex.: 18,022,006 is eighteen million, twenty-two thousand, six. (the unit group isn't usually named).

To switch from words into place value identify the period names first. I recommend drawing sets of underlines for each period as a first step.

Ex.: Seven thousand, five hundred seven is 7 , 5 0 7 .

Note: the commas in the word form are in the same places as in the place value form.

Problems for class use:

Write in word form:

1) 909

2) 25,031

3) 750,001

4) 2,013,504

Write in place value form:

5) One hundred twenty-three thousand, five.

6) Forty-nine thousand, four hundred fifteen.

7) Seventeen million, seven hundred thousand, seven

Date: ______________

Take extra notes here:

Place examples here:

Learning Tip:
Don't let the few minutes you are sitting in the classroom before class go to waste! Ask questions of your neighbor, review your notes, and get in a "math" frame of mind!

Objectives on this page: Adding whole numbers
Finding perimeter

Definitions: addends – things that are added
sum – the answer after adding
additive Identity – the property that adding zero does not change anything
Associative Law of Addition – the idea that the grouping of numbers in addition can vary without changing the answer. That is, $a + (b + c) = (a + b) + c$
Commutative Law of Addition – the idea that changing the order of an addition problem does not change the answer. That is, $a + b = b + a$
polygon – a closed 2-dimensional figure consisting of straight line segments
perimeter – the distance around a figure

When adding, be sure to line up the problem correctly so that you are adding like things! Add ones with ones, tens with tens, and hundreds with hundreds to avoid errors.

To find the perimeter of a polygon, add the lengths of the sides.
Note: look for symmetry – not all sides will be given!

Problems for class use:

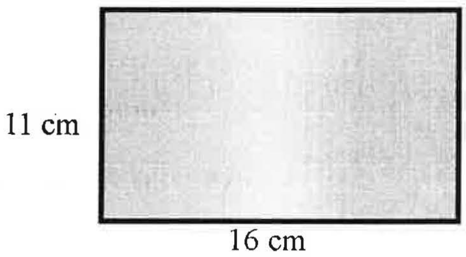
Find the sums:

1) 783 + 5703 + 529

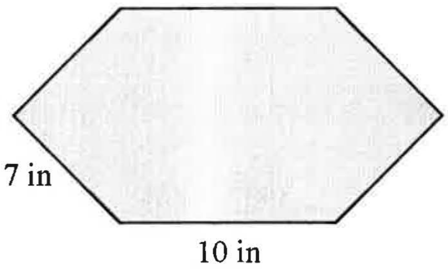
2) 10,001 + 9090 + 888 + 2000

Find the perimeters (make sure to include the units in the answer):

3)



4)



Date: ____________

Take extra notes here:

Place examples here:

Learning Tip:
Daydreaming = not learning. If you catch yourself not paying attention in class, increase the amount of writing you are doing in your notes or work some sample probems to keep alert!

Objectives on this page: Converting between addition and subtraction sentences
Subtracting whole numbers

Definitions: minuend – the number you start with in a subtraction problem
subtrahend – the number you subtract in a subtraction problem
difference – the answer after subtracting

Addition and subtraction are opposites of each other. So, if you have a problem (sentence) of one type, you can rearrange it to make a sentence of the other type.

Ex.: If we are given the sentence "8 - 3 = 5", a related addition sentence is "5 + 3 = 8".

Subtracting also, requires that you line up the problem correctly so that you are adding like things! Add ones with ones, tens with tens, and hundreds with hundreds to avoid errors.

Note: When "borrowing", making a place value of the minuend one less means you have ten more in the next place value. This is because we use a base 10 system.

Problems for class use:

Find a related sentence for:

1) 12 – 4 = 8

2) 25 - 2 = 23

3) 4 + 7 = 11

4) Find two related sentences for 6 + 1 = 7

Find the difference:

5) 533 - 54

6) 783 - 529

7) 10,001 - 9090

Date: ____________

Take extra notes here:

Place examples here:

Learning Tip:
Don't assume that you can figure something out on your own later. If you are confused on a point, make a note of it in your notes and ask the professor, a tutor, or a classmate about it before you go home!

Objectives on this page: To round to the nearest 10, 100, or 1000

To round whole numbers (a bit different than the text):

1) Underline the indicated place value and all digits to its left
2) Mark the indicated place value and cover all digits to its right
3) If the digit to the right of the mark is a 5, 6, 7, 8, or 9, increase the underlined number by 1; otherwise do nothing.
4) Change all digits not underlined to zeros.

Suggestion: Circle the digit to the right of the mark so that its clear which digit to work with.

Ex.: Round 4321 to the nearest hundred.

$\underline{43}$②1

↑ ↖not 5, 6, 7, 8, or 9, so we do nothing

hundreds

↙ Two digits (2 and 1) were not underlined, so we replace them with two zeros

4321 rounds to 4300

Problems for class use:

Round to the indicated place value:

1) 785 (ten)

2) 3,405 (hundred)

3) 9,987 (thousand)

4) 9,987 (hundred)

Date: ____________

Take extra notes here:

Place examples here:

Learning Tip:
The single best thing you can do for yourself in a math class is to find a study companion or two. Math is learned best when there is someone who can immediately help when you get stuck.

Objectives on this page: To estimate sums and differences by rounding
To compare whole numbers

Frequently you will need to get fairly close, fairly fast. To simplify the calculations, we use rounding. This process is called estimating.

To find an estimate for an addition or subtraction problem:

1) Find the place value stated in the problem. If no place value is stated, use the greatest place value in the problem.
2) Round all numbers to the place value you found in step 1).
3) Add or subtract the rounded values.

To compare whole numbers:

1) Find the highest place value of each number
2) If one number has a higher place value, it is larger
3) If both numbers have the same largest place value, look at which has the largest digit
4) If both numbers have the same digit, go to the next largest place value and repeat steps 3 & 4 as needed

---

Problems for class use:

Estimate the sums by rounding to the nearest hundred:

1) 533 + 254

2) 783 + 5703 + 429

Estimate the differences by rounding to the nearest ten:

3) 533 - 54

4) 783 - 529

Compare the numbers, writing "<" or ">" in the space provided:

5) 25 ___ 31

6) 750 ___ 9

7) 99 ___ 500

Date: ____________

Take extra notes here:

Place examples here:

Learning Tip:
Class time is valuable learning time. Although you will often need to ask someone questions later, it is the professor's best effort to explain what is going on in the class. Make sure you attend and pay attention!

Objectives on this page: To multiply whole numbers

Definitions: factors – things that are multiplied
product – the answer after multiplying
Multiplicative Identity – the idea that multiplying by one does not change anything
Associative Law of Multiplication – the idea that the grouping of numbers in multiplication can vary without changing the answer. That is, a • (b • c) = (a • b) • c.
Commutative Law of Multiplication – the idea that changing the order of a multiplication problem does not change the answer. That is, a • b = b • a.
Distributive Law – a • (b + c) = a • b + a • c

To solve multiplication problems, one method is:

1) Expand each number.
2) Multiply every possible combination of digits in the first number with the digits in the second number.
3) Add the results to get the final product.

Problems for class use:

Find the product (ask whether you should use the expanded method described above before you start):

1) 652 x 6

2) 328 x 52

3) In the problem 12 • 6 = 72, the number 12 is called a ____________________, the number 6 is called a ____________________, and the number 72 is called the ____________________.

4) The fact that 7 • 4 = 4 • 7 is an example of the ____________________________________.

5) Using the numbers 2, 3, and 4, give an example of the Distributive Law. Then solve both sides of your equation to show that the law is in fact true.

Date: ____________

Take extra notes here:

Place examples here:

Learning Tip:
Your professor has probably taught your course many times. If he makes mention that a point is important or commonly confused, make sure you highlight that point in your notes!

Objectives on this page: To estimate products by rounding

Frequently you will need to get fairly close, fairly fast. To simplify the calculations, we use rounding. This process is called estimating.

To find an estimate for a multiplication problem:

1) Round each number as specified. If no place value is specified, round each individually to its largest place value.
2) Multiply the rounded numbers.

Ex.: Estimate 6110 x 87 by rounding both numbers to the nearest hundred.

Problems for class use:

Estimate the products by rounding to the nearest ten:

1) 33 • 28

2) 783 • 5703

Estimate the products by rounding to the nearest hundred:

3) 533 • 264

4) 783 • 5629

5) 722 • 118

6) 2008 • 1977

Date: ______________

Take extra notes here:

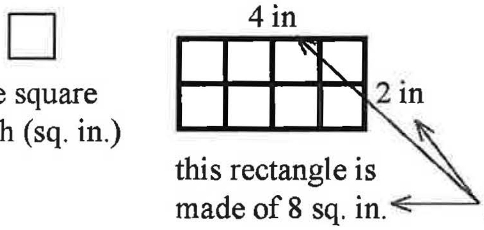
Place examples here:

Learning Tip:
Even the best notetaking systems have to be individualized. Record the details of the class, your trouble spots, and other important information specific to your course. Make sure you take notes!

Objectives on this page: To find the area of a rectangle (or square)

Definition: area – the amount of surface within a figure

Area refers to the amount of surface within a two-dimensional figure. It is always measured in square units.



To find the area of a rectangle, multiply the base (length) by the height.

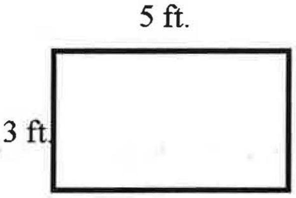
Note that squares are a special kind of rectangle, so the same rule applies.

Note that you must have the <u>same</u> types of units in both directions!

Problems for class use:

1): Find the area of a rectangle with a base of 12 in. and a height of 8 in.

2) Find the area of the given rectangle:



3) Find the area of a rectangle with a base of 7 in. and a height of 2 ft.. (Note that 12 in. = 1 ft..)

4) Find the area of a square with a base of 5 cm. and a height of 5 cm..

Date: ____________

Take extra notes here:

Place examples here:

Learning Tip:
Flashcards are a great way to learn anything that requires memorization. Place the hint on one side of the card and whatever you need to learn on the other. Then quiz yourself when you have a spare minute.

Objectives on this page: To convert between multiplication and division sentences

Just as addition and subtraction are opposites, multiplication and division are opposite operations. So, if you have a problem (sentence) of one type, you can rearrange it to make a sentence of the other type.

To create a related multipication sentence for a division sentence, use the same numbers in the reverse order and add "×" and "=" signs to make the sentence true.

To create a related division sentence for a multiplication sentence, use the same numbers in another order and add "÷" and "=" signs to make the sentence true.

Note: There are two different division sentences for each multiplication sentence.

Ex.: If we are given the sentence "6 ÷ 3 = 2", a related multiplication sentence is "2 • 3 = 6".

Problems for class use:

Find a related sentence for:

1) 12 ÷ 4 = 3

2) 24 ÷ 2 = 12

3) 28 ÷ 7 = 4

Find two related sentences for:

4) 6 • 3 = 18

5) 5 • 12 = 60

Date: ______________

Take extra notes here:

Place examples here:

Learning Tip:
Some people like to rewrite their notes in preparation for a test. A better approach is to revise your notes. Take the time to look over your notes thus far. If there's anything that's unclear, go ask a tutor!

Objectives on this page: To find the quotient of whole numbers

Definitions:
dividend – the number being divided
divisor – the number we are dividing by
quotient – the answer to a division problem
partial quotient – the whole number part of an answer
remainder – any part that does not divide evenly

The traditional long division method is not easy to describe in words. However, the same four steps are used over and over.

A way to remember! →

Determine the digit you will use in the quotient	Daddy
Multiply the digit by the divisor	Mother
Subtract this product from the place value in the dividend	Sister
Bring down the next digit from the dividend and repeat this process	Brother

Note that division problems can be written in long division form, ÷ (operator) form, or fractional form. The following are all the same problem:

$14\overline{)243}$ $243 \div 14$ $\frac{243}{14}$

Problems for class use:

Find the divisor:

1) **$55 \div 3$**

2) **$4207 \div 7$**

3) **$6052 \div 60$**

4) **$3281 \div 52$** *(Answer: 63r5)*

Date: ________________

Take extra notes here:

Place examples here:

Learning Tip:
Nearly everything covered in a class will be on at least one test and the final exam. Don't hope that a topic won't be on the test. Your studying needs to be sufficient so that you know how to do everything!

Objectives on this page: To solve applications using a strategy

Solving application (word) problems is often a lot harder than solving problems that have already been set up for you. In 1945, George Polya wrote the text "How To Solve It", and many strategies after that date have been based on Polya's work.

To solve an application problem:

1) **Understand what is being asked (Familiarize)**
2) **Devise a plan (Translate into math)**
3) **Carry out the plan (Solve)**
4) **Check your answer (Check)**
5) **State your answer in sentence form (State)**

Common key words [and situations] that often suggest:

addition -- sum, total, increased, more
subtraction -- difference, change, how much ___ (more, less, bigger, etc.), reduce
multiplication -- product, of, times, array, [rectangular arrangements], [repeated addition]
division -- quotient, [repeated subtraction], [dividing into equal groups]

Problems for class use:

1) **What can one do to familiarize oneself with an application problem?**

2) **If it is not immediately obvious, what can one do to help devise a plan?**

3) **How can one check an answer? State at least two ways.**

4) **It is traditional to always state word problems in complete sentence form. What advantage is there in writing your answer as a sentence?**

Date: ____________________

Take extra notes here:

Place examples here:

Learning Tip:
Taking notes is important, but so is paying attention. If you are spending all your class time copying and you are unable to listen to the instructor, ask the instructor to suggest ways to lighten your writing load.

Objectives on this page: To write exponential notation
To evaluate exponents

Defintions: exponential notation – a short notation for writing repeated multiplication
base – the number that is repeatedly multiplied
exponent – the number of times the base is multiplied
squared – to multiply the base by itself
cubed – to multiply the base by itself and again by itself

Two times we have written "5"

Multiplication can be thought of as repeated addition: $2 \cdot 5 = 5 + 5 = 10$.

Exponents can be thought of as repeated multiplication: $2^5 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$

The base is the number we work with

The exponent shows how many copies of the base

Ex.: Write exponential notation for $4 \cdot 4$

$4 \cdot 4 = 4^2$

Ex.: Evaluate 3^3

$3^3 = 3 \cdot 3 \cdot 3$
$= 9 \cdot 3$
$= 27$

Problems for class use:

Write in exponential notation:

1) $6 \cdot 6 \cdot 6$

2) $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

3) $1 \cdot 1 \cdot 1 \cdot 1$

4) 9

Evaluate:

5) 5^2

6) 4^3

7) 3^4

8) 7^1

Date: ____________

Take extra notes here:

Place examples here:

Learning Tip:
Have you been to office hours yet? A professor who knows you personally is a benefit if you have a complaint or challenge a score. You can also ask your professor for help on a topic while you're there.

Objectives on this page: To find the arithmetic mean (average) of a set of numbers

Definition: <u>mean</u> -- a type of average found by adding values and then dividing by the number of values

When we say the word “average”, we mean “typical” or “normal”. There are several different “averages” one can find using mathematics, but the most used is the arithmetic mean. Although it isn’t quite correct, when someone says “average” they usually want you to find the arithmetic mean.

To find the arithmetic mean of a set of values:

1) **Add the values.**
2) **Divide the total you found by the number of values.**

Ex.: Find the arithmetic mean of 1, 1, 4, 7, and 12.

$$\begin{aligned} \text{mean} &= (1 + 1 + 4 + 7 + 12) \div 5 \\ &= 25 \div 5 \\ &= 5 \end{aligned}$$

First we add the values (you have to write the parentheses too!)

Then we divide by five since there are five numbers

Problems for class use:

Find the arithmetic mean (average) of:

1) 3, 5, 5, 6, and 6

2) 22, 33, 9, and 4

3) 32, 8, 8, 3, and 4

4) 21, 28, and 32

Date: ____________________

Take extra notes here:

Place examples here:

Objectives on this page: To solve multiple step problems using the order of operations

Although we read from left to right, there are reasons why we don't do this in math. Math is usually done from the most complex to the least complex operation.

To solve a multiple part problem:

1A) Solve everything in Parentheses, brackets or parts of fractions using steps 2, 3, & 4
1B) Do the rest of the Problem, using steps 2, 3, & 4
2) Simplify all Exponents
3) Multiply and Divide going from left to right.
4) Add and Subtract going from left to right.

Mnemonic technique: Please Excuse My Dear Aunt Sally!

Ex.: $3 \cdot 4 + 5$
$= 12 + 5 \leftarrow$ Multiply before adding! $\rightarrow = 3 + 9$
$= 17$

Ex.: $3 + 4 \cdot 5$
$= 3 + 9$
$= 12$

Problems for class use:

Simplify, using the order of operations:

1) $25 - 3 \cdot 7$

2) $56 - (17 + 7)$

3) $3^5 + 7^2$

4) $3^2 + 7 - 2\,(3)$ ← A pair of numbers without anything but parentheses between them should be multiplied.

5) $5 - 4 \div 2^2$

6) $13 + 9 \cdot (2^3 \cdot 3 - 4) \div 4$

Date: ______________

Take extra notes here:

Place examples here:

Objectives on this page: To find the natural factors of a number

Definitions: factors -- numbers that are multiplied together
divisor -- a number that divides a given number exactly (without a remainder)

To find the factors of a given number:

1) **Make a list of numbers from one up to the number which when squared is at least as large as the given number.**
2) **Test each number on the list to see if it is a divisor of the given number.**
3) **If the number is a divisor, then it is a factor. The given number divided by the factor is also a factor.**

Ex.: Find all the factors of 14.

1 x 14
2 x 7
~~3~~ *(14 ÷ 3 has a remainder)*

For this problem, we can stop checking for factors at 3 since the next number (4) when squared ($4^2 = 16$) is larger than 14. This is how we know when we can stop!

The factors of 14 are 1, 2, 7, and 14.

Problems for class use:

List all the factors of:

1) 18

2) 24

3) 50

4) 54

5) 68

6) 99

Date: ____________________

Take extra notes here:

Place examples here:

Objectives on this page: To find some of the multiples of a number
To determine if a number is divisible by another

Definitions: multiple – is the product of another number and any natural number
divisible – can be divided by another number with no remainder

The word multiple deals with multiplying. The first multiple of a number is one times that number, the second multiple of a number is two times a number and so on.

Ex.: The first five multiples of 44 are 44, 88, 132, 176, and 220, which equal 44 • 1, 44 • 2, 44 • 3, 44 • 4, and 44 • 5 respectively.

The words "divisor" and "factor" and the words "divisible" and "multiple" are closely related. Thus, we can ask a question in four equivalent ways. Of these, "divisible" is usually the easiest.

Ex. Is 42 divisible by 3?
Is 42 a multiple of 3?
Is 3 a divisor of 42?
Is 3 a factor of 42?

Since 42 ÷ 3 = 14r0, the answer is "yes".

These are all solved the same way. We will divide 42 by 3 to see the remainder. If the remainder is "0", the answer is "yes". If the remainder is not "0", the answer is "no".

Problems for class use:

Find the first five multiples of:

1) 4

2) 16

Determine by using long division:

3) Is 648 a multiple of 4?

4) Is 6 a factor of 424?

5) Is 7 a divisor of 770?

6) **Is 663 divisible by 13?**

Date: ____________________

Take extra notes here:

Place examples here:

Objectives on this page: To determine whether a number is prime, composite, or neither

Definitions: prime – a natural number with exactly two natural factors
composite – a natural number with more than two natural factors

Numbers are considered prime if they have two factors precisely. The word composite is used if there are more than two factors. Some numbers, like 0 and 1 are neither prime nor composite.

To prove whether a number is prime or composite:

1) **Use the method we learned to find factors**
2) **If a factor other than 1 or the number itself is found, it is composite. If not, it is prime.**

Ex.: Is 99 prime or composite? ← We will look for factors. 1 and 99 are two factors -- if there are any more, then 99 will be composite. If 1 and 99 are the only two factors, then 99 will be prime.

1 x 99
~~2~~ *(99 ÷ 2 has a remainder)*
3 x 33 ← We found a third factor (3), so 99 is composite. Notice that we do not have to keep checking once we find a third factor.
4 x ...
COMPOSITE

Problems for class use:

Prove whether or not the number is prime or composite:

1) 21

2) 41

3) 61

4) 81

5) 101

6) If we were deciding whether or not 899 was a prime number, we would have to check 1, 2, 3, ... to see if they were factors. At what number could we stop checking? Why?

Date: ____________________

Take extra notes here:

Place examples here:

Objectives on this page: To find the prime factorization of a composite number

Definitions: factorization -- a number written as a multiplication problem
prime factorization -- a number written as a multiplication problem consisting of only prime factors

There are several ways to find a prime factorization. Only the "factor tree" method is mentioned here, so ask your instructor if you wish to use a different technique.

To write the prime factorization of a number using the "factor tree" method:

1) **Factor the desired number in any way that does not use "1".**
2) **If any factor is a prime number, circle it.**
3) **If any factor is not a prime number, repeat these steps with the factor.**
4) **Write down a multiplication problem using the circled factors as your answer.**

To memorize:

"The first five primes are 2, 3, 5, 7, and 11."

Problems for class use:

Find the prime factorization of:

1) 28

2) 63

3) 78

4) 100

Date: ____________

Take extra notes here:

Place examples here:

Objectives on this page: To quickly test whether a number is divisible by 2, 5,10, 3 or 9

Definitions: divisibility test – any technique for finding whether one number is divisible by another

We will need to check whether one number is divisible by another quite frequently in this course. A few numbers have shortcut tests.

Divisibilty Test #1 (look at the last digit on the right):

A number is divisible by 2 if the last digit is 0, 2, 4, 6, or 8.
A number is divisible by 5 if the last digit is 0 or 5.
A number is divisible by 10 if the last digit is 0.

Divisibility Test #2

A number is divisible by 3 if the digit sum is a multiple of 3.
A number is divisible by 9 if the digit sum is a multiple of 9.

For any number, remember that you can test for divisibility by dividing. If the remainder is zero, then the number is divisible. If the remainder is not zero, then the number is not divisible.

Problems for class use:

Which of 24, 9,600, and 495 are:

1) divisible by 2?

2) divisible by 3?

3) divisible by 5?

4) divisible by 10?

5) Without actually doing the division, determine whether or not 9, 534, 699 is divisible by 9.

Date: ____________

Take extra notes here:

Place examples here:

Objectives on this page: To write fraction notation for a set of objects

Definitions:
fraction -- a division problem
numerator -- the top number (dividend) of a fraction
denominator -- the bottom number (divisor) of a fraction

If the parts of a set are equal in value or size, then a fraction is the number of parts that we wish to count over the number of parts it takes to make one entire group. That is:

$$\frac{\text{number of parts with the quality we want}}{\text{number of parts needed to make one whole group}}$$

Note the denominator makes one whole group -- this doesn't have to be the entire set!

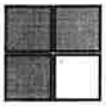
Ex. What fraction of the figure is shaded?

$$\frac{13}{6}\begin{array}{l}\leftarrow \text{13 shaded objects}\\ \leftarrow \text{6 objects to make a whole group}\end{array}$$

Problems for class use:

What fraction of the figure is shaded?

1)



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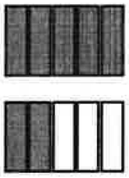

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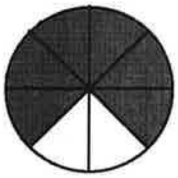

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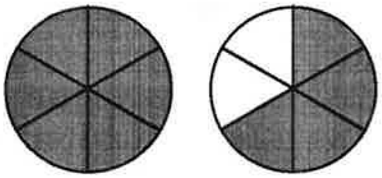

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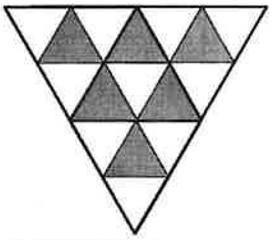

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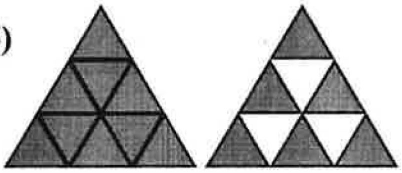

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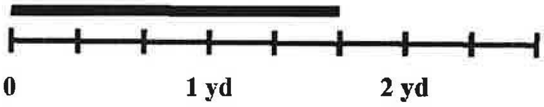

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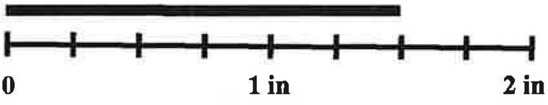


Date: ____________________

Take extra notes here:

Place examples here:

Objectives on this page: Adding like fractions
Subtracting like fractions

Definitions: like fractions – fractions with common denominators
unlike fractions – fractions with different denominators

One can only add or subtract like things. Two apples plus three apples makes five apples. Two oranges plus three oranges makes five oranges. Two apples plus three oranges makes _____? (We can't compare apples and oranges -- they are unlike each other.)

Fractions are like each other when they have the same denominator. These are the only kind of fraction we will add or subtract right now.

To add or subtract like fractions:

1) **Add or subtract the numerators of the fractions**
2) **Write the answer you found in step 1) as the numerator of the answer**
3) **Write the common denominator as the denominator of the answer**
4) **Simplify, if possible (We will learn how to do this later.)**

Problems for class use:

Add:

1) $\frac{3}{8} + \frac{4}{8}$

2) $\frac{2}{3} + \frac{2}{3}$

3) $\frac{1}{5} + \frac{4}{5}$

4) $\frac{3}{10} + \frac{5}{10}$

5) $\frac{7}{11} + \frac{3}{11}$

6) $\frac{2}{2} + \frac{3}{2}$

Subtract:

7) $\frac{11}{6} - \frac{4}{6}$

8) $\frac{13}{15} - \frac{7}{15}$

9) $\frac{4}{5} - \frac{1}{5}$

Date: ____________________

Take extra notes here:

Place examples here:

Objectives on this page: To simplify special fractions
To multiply a whole number by a fraction

A few fractions are worth memorizing:

$\frac{n}{n}$ = 1, where n is any number besides zero

$\frac{n}{1}$ = n, where n is any number

$\frac{n}{0}$ is undefined, where n is any number

$\frac{0}{n}$ = 0, where n is any number besides zero

Since multiplying is repeated addition, $4 \times \frac{2}{5}$ is the same as $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{8}{5}$
Therefore, we can multiply natural numbers by fractions by just repeatedly adding the fractions.

Problems for class use:

Simplify:

1) $\frac{6}{1}$ **2)** $\frac{4}{4}$ **3)** $\frac{23}{1}$ **4)** $\frac{0}{9}$

5) $\frac{9}{0}$ **6)** $\frac{30}{30}$ **7)** $\frac{7}{7-7}$ **8)** $\frac{12}{0+1}$

Use repeated addition to multiply:

9) $2 \bullet \frac{4}{5}$ **10)** $4 \bullet \frac{4}{15}$ **11)** $3 \bullet \frac{1}{4}$

Date: ____________________

Take extra notes here:

Place examples here:

Objectives on this page: To find equivalent fractions to a given fraction

Definitions: equivalent fractions -- fractions that represent the same amount

To make an equivalent fraction given the new denominator:

1) **Find what you have to multiply the first denominator by to equal the second (new) denominator.**
2) **Multiply the first numerator by the value you found in step 1).**

Ex.: $\frac{2}{9} = \frac{x}{36}$.

It is o.k. to leave this line out of your work

$\frac{2}{9} \cdot 1 = \frac{x}{36}$ (since multiplying by one doesn't change a result)

$\frac{2}{9} \cdot \frac{4}{4} = \frac{x}{36}$ (since 9 times 4 is what equals 36)

$\frac{2}{9} = \frac{8}{36}$

Problems for class use:

Use multiplying by one to find an equivalent fraction:

1) $\frac{3}{4} = \frac{x}{8}$

2) $\frac{2}{8} = \frac{x}{40}$

3) $\frac{5}{6} = \frac{x}{24}$

4) $\frac{3}{10} = \frac{x}{30}$

5) $\frac{7}{16} = \frac{x}{96}$

6) $\frac{1}{2} = \frac{x}{20}$

7) $\frac{5}{6} = \frac{x}{48}$

8) $\frac{3}{5} = \frac{x}{45}$

9) $\frac{4}{7} = \frac{x}{84}$

Date: ____________________

Take extra notes here:

Place examples here:

Objectives on this page: To completely simplify a fraction

Definitions: completely simplified fraction -- a fraction whose numerator and denominator have no common factors other than one

There are several different ways one can completely simplify a fraction. Due to space limitations, only one technique is stated here. Be sure to ask what other techniques exist.

To simplify fractions (prime factorization technique):

1) **Write both the numerator and denominator in prime factorization form.**
2) **Cancel any factors that are identical in both the numerator and denominator.**
3) **Multiply together the remaining factors in the numerator to get the numerator's answer. Do the same for the denominator.**
   Note: If all factors cancel, write "1", not "0"!

Problems for class use:

Simplify:

1) $\frac{12}{16}$

2) $\frac{24}{30}$

3) $\frac{24}{80}$

4) $\frac{42}{96}$

5) $\frac{12}{90}$

6) $\frac{80}{18}$

7) $\frac{34}{102}$

8) $\frac{425}{650}$

9) $\frac{180}{420}$

Date: ____________________

Take extra notes here:

Place examples here:

Objectives on this page: To multiply fractions

Since multiplying is repeated addition, 4 x $\frac{2}{5}$ is the same as $\frac{2}{5} + \frac{2}{5} + \frac{2}{5} + \frac{2}{5} = \frac{8}{5}$
But 4 x 2 = 8, which gives a hint on how to multiply fractions in general.

To multiply fractions and mixed numbers:

1) **Convert each number to fraction form.**
2) **Simplify by cancelling any factors that are identical in both the numerator and the denominator.**
3) **Multiply the remaining factors in the numerator to obtain the numerator of the answer. Do the same for the denominator.**
4) **Completely simplify the answer.**

Ex.: Find $\frac{10}{15} \bullet \frac{6}{4}$.

$$\frac{6}{15} \bullet \frac{6}{4} = \frac{\cancel{2} \bullet \cancel{3}}{\cancel{3} \bullet 5} \bullet \frac{\cancel{2} \bullet 3}{\cancel{2} \bullet \cancel{2}}$$ (notice that only top/bottom pairs may be cancelled)

$$= \frac{3}{5}$$

Problems for class use:

Multiply (and simplify):

1) $\frac{4}{9} \bullet \frac{21}{16}$

2) $\frac{8}{2} \bullet \frac{9}{24}$

3) $\frac{8}{16} \bullet \frac{8}{18}$

4) $\frac{24}{15} \bullet \frac{45}{32}$

5) $\frac{25}{24} \bullet \frac{12}{60}$

6) $\frac{4}{20} \bullet \frac{5}{20}$

Date: ____________

Take extra notes here:

Place examples here:

Objectives on this page: To find the reciprocal of a fraction
To divide fractions

Definition: reciprocal (practical definition) -- an operation on a fraction that makes the numerator the denominator and vice-versa.

To find the reciprocal of a number:

1) **Convert each number to fraction form.**
2) **Switch the numerator with the denominator and vice-versa.**

To divide fractions and mixed numbers:

1) **Convert each number to fraction form.**
2) **Convert the problem to a multiplication problem switching the divisor (the second fraction) with its reciprocal.**
3) **Simplify by cancelling any factors that are identical in both the numerator and the denominator.**
4) **Multiply the remaining factors in the numerator to obtain the numerator of the answer. Do the same for the denominator.**
5) **Completely simplify the answer.**

Problems for class use:

Divide (and simplify):

1) $\frac{4}{9} \div \frac{16}{21}$

2) $\frac{8}{6} \div \frac{9}{24}$

3) $\frac{8}{16} \div \frac{8}{18}$

4) $\frac{24}{15} \div \frac{48}{45}$

5) $\frac{25}{24} \div \frac{15}{60}$

6) $\frac{4}{20} \div \frac{5}{20}$

Date: ____________________

Take extra notes here:

Place examples here:

Objectives on this page: To compare fractions using the means-extremes rule

To compare two fractions:

1) **Write the fractions side-by-side.**
2) **Multiply the left denominator by the right numerator and write the product over the right fraction.**
3) **Multiply the right denominator by the left numerator and write the product over the left fraction.**
4) **Whichever fraction has the greater product is the greater fraction.**

Ex.: Which is larger, $\frac{3}{5}$ or $\frac{7}{12}$?

Since 36 > 35,

$\frac{3}{5} > \frac{7}{12}$.

Problems for class use:

Determine which fraction is larger:

1) $\frac{6}{9}$ ___ $\frac{12}{16}$

2) $\frac{8}{9}$ ___ $\frac{21}{24}$

3) $\frac{5}{16}$ ___ $\frac{6}{18}$

4) $\frac{24}{45}$ ___ $\frac{6}{11}$

5) $\frac{5}{24}$ ___ $\frac{11}{60}$

6) $\frac{1}{4}$ ___ $\frac{1}{5}$

Date: ______________

Take extra notes here:

Place examples here:

Objectives on this page: To find the least common multiple (L.C.M.) of a set of numbers

Definition: Least Common Multiple (L.C.M.) -- The smallest number that is a multiple of each of two or more given numbers.

To find the L.C.M. of a set of numbers:

1) **Find the prime factorization of each number.**
2) **Find the prime factorization with the <u>most</u> 2s and write down that many 2s.**
3) **Repeat step 2) with 3, 5, 7, and any other prime numbers that show up in the prime factorizations.**
4) **Multiply together the prime numbers you wrote down.**

Ex.: Find the L.C.M. of 24 and 30.

$24 = \underline{2} \bullet \underline{2} \bullet \underline{2} \bullet 3$ ← First, find the prime factorization of each number.

$30 = 2 \bullet 3 \bullet \underline{5}$

Then, write down the most repetitions of each prime found in our prime factorizations

$\text{L.C.M.} = 2 \bullet 2 \bullet 2 \bullet 3 \bullet 5$ In the case of a tie, only list the primes once

$= 120$

Problems for class use:

Find the L.C.M. of:

1) 8 and 14

2) 15 and 12

3) 40 and 50

4) 18 and 30

5) 35 and 105

6) 36, 40, and 44

Date: ____________

Take extra notes here:

Place examples here:

Objectives on this page: To convert fractions to equivalent like fractions

To finish this chapter, we will combine several objectives from earlier in the course. You may wish to review prime factorization, equivalent fractions, and least common multiple (L.C.M.) before attempting the problems on this page.

To convert fractions to equivalent like fractons:

1) **Find the L.C.M. of the denominators.**
2) **Find what you have to multiply the first denominator by to equal the L.C.M..**
3) **Multiply the first numerator by the value you found in step 2).**
4) **Repeat steps 2) & 3) with the second fraction.**

Problems for class use:

Convert the fractions to equivalent like fractions:

1) $\frac{5}{9}$ **&** $\frac{5}{6}$

2) $\frac{7}{20}$ **&** $\frac{1}{8}$

3) $\frac{5}{16}$ **&** $\frac{7}{10}$

4) $\frac{7}{16}$ **&** $\frac{11}{12}$

5) $\frac{4}{25}$ **&** $\frac{11}{35}$

6) $\frac{1}{24}$ **&** $\frac{1}{18}$

Date: ____________

Take extra notes here:

Place examples here:

addends – things that are added
Additive identity – the property that adding zero does not change anything
approximately equal ($\approx$) – nearly the same
area – the amount of surface within a figure
average – a single value representing a set of data (see <u>mean</u>)
base – the number that is repeatedly multiplied
Associative Law of Addition – the idea that the grouping of numbers in addition can vary without changing the answer. That is, $a + (b + c) = (a + b) + c$
Associative Law of Multiplication – the idea that the grouping of numbers in multiplication can vary without changing the answer. That is, $a \bullet (b \bullet c) = (a \bullet b) \bullet c$
Commutative Law of Addition – the idea that changing the order of an addition problem does not change the answer. That is, $a + b = b + a$
Commutative Law of Multiplication – the idea thatchanging the order of a multiplication problem does not change the answer. That is, $a \bullet b = b \bullet a$
completely simplified fraction – a fraction whose numerator and denominator have no common factors other than one
composite – a natural number with more than two natural factors
cubed – to multiply the base by itself and again by itself
denominator – the bottom number (divisor) of a fraction
difference – the answer after subtracting
digits – 0,1,2,3,4,5,6,7,8,9
Distributive Law – $a \bullet (b + c) = a \bullet b + a \bullet c$
dividend – the number being divided
divisibility test – any technique for finding whether one number is divisible by another
divisible – can be divided by another number with no remainder
divisor^1 – the number we are dividing by
divisor^2 – a number that divides a given number exactly (without a remainder)
equal (=) – the same quantity
equivalent fractions – fractions that represent the same amount
expanded notation – a form of writing numbers where the place value of each digit is written out
exponent – the number of times the base is multiplied
exponential notation – a short notation for writing repeated multiplication
factorization – a number written as a multiplication problem
factors^1 – things that are multiplied
factors^2 – numbers that are multiplied together
fraction – a division problem
greater than (>) – being of a larger amount, as in $8 > 3$
least common multiple (L.C.M.) – the smallest number that is a multiple of each of two or more given numbers
less than (<) – being of a smaller amount
like fractions – fractions with common denominators (the denominators are the same)
mean (arithmetic) – a type of average found by adding values and then dividing by the number of values
minuend – the number you start with in a subtraction problem
multiple – the product of a given number and any natural number
Multiplicative identity – the property that multiplying by one does not change anything
Natural (counting) numbers – $\{1, 2, 3, 4, 5, \ldots\}$
not equal ($\neq$) – not the same quantity

LERN 48 Glossary (continued)

numerator – the top number (dividend) of a fraction
partial quotient – the whole number part of an answer
perfect square – a number that equals a factor times itself (1, 4, 9, 16, 25, … are perfect squares since they equal 1 • 1, 2 • 2, 3 • 3, 4 • 4, 5 • 5, and so forth)
perimeter – the distance around a figure
periods – groups of three digits used in place value notation
polygon – a closed 2-dimensional figure consisting of straight line segments
prime – a natural number with exactly two natural factors
prime factorization – a number written as a multiplication problem consisting of only prime factors
product – the answer after multiplying
quotient – the answer to a division problem
reciprocal (practical definition) – an operation on a fraction that makes the numerator the denominator and vice-versa.
remainder – any part that does not divide evenly
squared – to multiply the base by itself
standard notation – a form of writing numbers where the value of each digit depends on its location in the number
subtrahend – the number you subtract in a subtraction problem
sum – the answer after adding
unlike fractions – fractions with different denominators
Whole numbers – the natural numbers and zero

Time Management Tips

People often talk about budgeting their money, but budgeting your time is just as important. Like it or not, you get 168 hours per week (24 x 7), so you had best use your time wisely. Here are some tips for maximizing your use of time:

Tip #1 – Sleep 49 – 56 hours a week

Typical teenagers (including 18 & 19 year olds!) need about 55 hours of sleep a week in order to function at their best. Older folks can get by with slightly less. If you get less than 50 hours of sleep a week, you will not be as alert, you will have a harder time learning new material, and you will make more "careless" errors. Although sleep can to some extent be "stored", you will do you best if you get 7 – 8 hours of sleep a night the same time every night. So set an alarm clock for the same time each morning and force yourself to go to sleep at the same time every night!

Tip #2 – Exercise at least a little

Exercise is known to have many beneficial effects. One of them is that it tends to make a person more alert and efficient for several hours after the exercise is over. Even a 20 minute brisk walk is enough to do the job. If you are getting enough sleep but still feel tired, it is probably due to too little exercise. So get some exercise each day!

Tip #3 – Leave lots of time for college

There's a reason why a student with 12 units is called "full-time". Twelve units are the equivalent to having a full-time job! Most colleges expect three hours of work per week per unit. That means that for a three unit class, you can expect to spend nine hours a week if you want a good grade! (This works out to about three hours in class, four or five hours doing the assignments, and one or two hours studying, reviewing your notes, etc.) Although you can "cut corners" in your first few classes and still pass, eventually you will come to wish you understood your earlier classes better. So plan on working two hours out of class for every hour you are in class!

Tip #4 – Don't over-commit yourself

If you add up all the hours you spend sleeping, getting ready in the morning, eating, and traveling from place to place, you will likely find that you have only about half of your 168 hours left to actually get anything done. Although it is possible to work 70 or 80 hours a week for brief periods of time, it is unrealistic to think you will be able to keep that pace all semester long. Take the number of units you are enrolled in and multiply it by three. Add to this the number of hours you work each week, including tasks like yardwork, housework, meetings you attend, and volunteer work. If this number is over sixty, cut back on something!

Tip #5 – Have some fun along the way, but make sure it's fun

All work and no play makes Jack a dull, bitter, twisted workaholic. So make sure you relax and play each week, but realize that your time is limited and valuable, so don't spend it on things you don't really want to do. Often times people will spend hours watching TV, surfing the Internet, or shopping at the mall. If that's what you really want to do, great! However, if there is something you want to do more, make certain that you do that first. There will never be enough time to do everything.

A Little Thing Called...
Anxiety

Everyone has experienced anxiety sometime in their lives. Usually it happens when we are about to try something new or when we are about to have a challenge like a race, a test, or maybe an interview. Many people experience things like "butterflies" in their stomachs, a faster heart rate, a fear that they wil fail, or an inability to concentrate. Some anxiety is good; it can actually help us to perform better. But too much anxiety is bad; it can interfere with our ability to concentrate or worse, it can keep us from trying at all.

There are some things that we can do to help us cope with feelings of anxiety. First, it always helps to practice or to study ahead of time. If you are going to an interiew, practice answering questions with a friend and review your resume. If you are going to run a race, practice every day and try to run under the same conditions. If you are going to take a test, start your studying at least several days in advance, and not just the day before. Succeeding when we practice gives us confidence when we are tested.

Second, make sure you're not too tired or hungry. Athletes and students who don't get a good night's sleep the night before a race or an exam tend to do poorly. After all, our brains use a huge amount of energy when they are thinking hard, and being tired makes both our bodies and brains shut down a bit. Having a good dinner the night before the test and having a good breakfast the morning of the test will help your body have energy, and if it comes beween a choice of studying late into the night or sleeping, it is usually better to get the sleep.. Again, you want to avoid having your body start to shut down before the race or the test is over.

Third, visualize doing well. Our bodies and our minds tend to follow hat we are thinking. For example, if an athlete looks at a jump and pretends that they are running up and sailing over the top, they will do a better job than an athlete who is thinking about not making the jump and falling over. If you find yourself going blank or feel your pulse racing, you should sit still for a moment and pretend that you are doing a perfect job. Closing your eyes and taking a couple of deep slow breaths can also help you stay calm.

Finally, if you are still nervous or are feeling anxious, realize that these feelings are normal. Everyone gets nervous before performing a task. Nervousness is a normal feeling and in some ways it can give you an edge and make you do better. Just remember to practice, sleep well, eat well, and visualize doing a perfect job.

A Little Thing Called...
Anxiety Questions

Please answer the following questions in complete sentences. You may use another sheet of paper if you wish.

1) Some authors have claimed that fully half of all students have math anxiety on a regular basis. How frequently do you feel anxious about math? What does math anxiety feel like to you?

2) Practicing well in advance of a test can lessen math anxiety. If you were told today that you would have a test in two weeks, what would you do to study for it? When would you do most of your studying?

3) It is well known that tired or hungry people don't do as well as rested well-fed people on math tests. How often do you feel tired in your classes? How often do you attend class without having yet eaten a meal that day?

4) The simple act of thinking you will fail a task makes it more likely that you will in fact fail. Describe (or make up) a situation where you observed someone making negative comments about himself. Then suggest what you think that person should have told himself.

Math Test Taking Tips

Although taking a test can be a stressful experience, it doesn't have to be. The best way to reduce stress is to be prepared. Here are some tips on how to do better on math tests.

BEFORE THE TEST

PRACTICE!

A lot of math anxiety is caused by thinking you will not be able to do the problems. Although you won't know the exact problems the professor will assign, you can get a pretty good idea of what to expect by reviewing your old tests and quizzes. (You did keep them, didn't you?) It may also help to review the "Chapter Test" or "Chapter Review" sections found at the end of the chapter in nearly all math books.

VERIFY!

You should not consider your studying complete until you can work all the types of problems you will be tested on. When a problem surfaces that you simply cannot figure out don't just hope the problem will not be on the test. Instead, see your professor during his office hours. This means that you have to start studying several days before the test so that you have time to ask questions and then study the newly relearned material.

REVIEW!

Reviewing your notes and reading the textbook are not the same as practicing. When you look over your notes, study the example problems, and read the comments you originally wrote, you are able to better see how the topics interact with and layer upon each other. This helps you get the "big picture" and is one of the reasons you took the notes to begin with.

SLEEP!

The nights before the test, it helps to schedule an hour or two of study time. However, you should still go to bed at your usual bedtime. Changing your bedtime throws off your body's natural rhythms, and staying up late to study the night before a test usually does more harm than good.

EAT!

The morning of the test, make sure you have breakfast, even if you usually don't eat it. Your brain needs glucose (a type of sugar) to work, and glucose is hard for your body to produce on an empty stomach. Fortunately, your body can easily convert the carbohydrates in practically any bread-like item into glucose, so practically any type of breakfast food works. Some people worry about caffeine, but the "jitters" only occur when you drink more than your usual amount of coffee. In fact, a little coffee is actually beneficial for most people!

ARRIVE!

Stress from being caught in a traffic jam, maneuvering through a full parking lot, or running across campus can increase your anxiety, so you do not want to be in a situation where you might be late for the test. Ideally, you should arrive a half-hour or so before the test time. Then, use that time to sit outside the building and just relax. Make sure you have a couple of pencils and whatever supplies you will need.

(Continued on the next page)

DURING THE TEST

AVOID DISTRACTIONS!
Lots of things can cause distractions – having to leave to use the restroom, having to get up to sharpen a pencil, a cell phone ringing, even a pretty girl. Try to plan ahead so that you are not as likely to be distracted. Use the restroom before the test, carry and extra pencil or two, and turn off your cell phone. If you are distracted by motion, consider sitting in the front of the class, and if you are distracted by sounds, consider sitting towards the sides of the classroom.

READ THE INSTRUCTIONS!
When you get the test, read the instructions carefully - even if what you should do seems obvious. Some instructors will insist on a certain format or will require that work be shown even on simple problems. Some problems may be stated in unusual ways. If you do not read the instructions for each and every problem, you are very likely to lose some points somewhere.

TAKE THE TEST FOUR TIMES!
The first time through a test, you should skim it, doing only the problems that you find short and easy. This is to give you some idea of the length of the test and to get you off to a good start. If you can answer a couple of questions quickly, you are less likely to feel anxious.

The second time through a test, do only the problems you know how to do. If you have any doubt about how to do them, skip them for now. Never take a test in order (i.e.: 1, 2, 3, …). The hardest problems are not always at the end!

The third time through a test, do the problems you don't know how to do. This might sound like strange advice, but math is a very logical subject, and if you think about a test question long enough, you can often figure out what the answer has to be. Even if you don't get the problem finished, write down what steps you can – most instructors give partial credit.

The fourth time through a test should take place when there are only 5 – 10 minutes remaining. Look over the remaining problems, and look for places where you think you might be able to get a few more points in a hurry. DON'T LEAVE EARLY AND DON'T GIVE UP! Your test isn't a race – it's an evaluation of what you can figure out in a set amount of time – yet people tend to panic when they see others turn in their tests. Keep your test until time is up, and perfect your answers as much as you can.

AFTER THE TEST

RELAX!
After the test, there isn't much you can do about your score, so don't second guess yourself. Now is the time to congratulate yourself for working hard and keeping up your good study habits. Of course, if your study habits were not that good... well, there's always the next test to study for.

KEEP IT!
Do not throw away a returned test. You can use your old tests to study for the final exam and to analyze what you need to study further. Make certain the professor added up your points correctly, and go over the problems you missed so that you understand what types of problems that you need to further practice.

Math Test Taking Tips Questions

Please answer the following questions in complete sentences. You may use another sheet of paper if you wish.

1) The author notes six things you can do before the test occurs. List these in what you believe to be the most to the least important. Why did you rank these tips in this order?

2) Re-read the section of the essy labeled "Take the Test Four Times". How does this technique compare with how you normally take math tests? If your current technique is similar, explain when and way you use it. If your current technique is not similar, explain how they differ.

3) Some people believe that there is no such thing as test-taking anxiety. Instead, these people argue that a student's anxiety comes not from the fact that he is taking a test, but rather that he is worried that he is underprepared. Thus, if you practice until the problem seem easy, you won't have much anxiety. Given the experience you've gained over the years, do you think that most test-taking anxiety comes from being underprepared? Be sure to support your answer.

Test Analysis Tips

In most sports, coaches will watch a recording of the game their team recently played. Often, they will have their players watch too. This does not occur because they are impressed with their work and want to bask in their own perfection. Rather, the games are viewed to that the coaches can determine what worked and what needs revision or further practice.

You probably don't have your own coach for math, but the same techniques that coaches use can benefit you. Instead of just looking at your test score and throwing the test away, you can analyze your test and use it to make yourself less likely to make the same mistakes again.

Tip #1 – Check the scoring

Even the most intelligent and dedicated professors make mistakes, and the first thing you should do upon receiving your returned test is to double check your professor's math. It's not unusual for a tired professor to accidentally take five points off of a four point problem or to make a mistake when adding up your score. Although this doesn't happen that frequently, it does happen, so always double check.

Tip #2 – Determine if miscommunication played a role

As you go through your test, make a note of places where you lost points for not showing sufficient steps or for not answering in the format desired. Although such lost points may be few, all points matter, and you don't want to keep getting lower scores just because you aren't stating your answers in the desired way. If your professor is deducting points for notation or formatting, he probably has his reasons, so be sure you understand how your work is expected to look.

Tip #3 – Note which sections or topics gave you trouble

Whenever you had points marked off, make a note of which section in your textbook the troublesome problem came from. Then, see if there are any trends. It frequently happens that most of your problems come from just a few sections of the textbook, and if you know which sections gave you trouble, then you know where you need to review.

Tip #4 – Determine what kind of errors you made

Go back through the test and make a note of the kinds of errors you made:

Careless errors exist when you know how to do the problem, but for some reason made a mistake.

Ex.:
$$\begin{array}{r} 32 \\ \underline{\times\ 4} \\ 78 \end{array}$$

(You know that 3 x 4 = 12, but for some reason wrote 7.)

Knowledge errors exist when you do not know how to do the problem or learned it wrong.

Ex.: $3 + 2^3$
$= 5^3$
$= 125$

(You forgot that exponents take priority over adding.)

Strategy errors exist when you do not know how to set up the problem, but could do the problem if it had been set up for you.

Ex.: What is the largest factor of 12?

(??? -- you just don't understand the question.)

(continued on the next page)

Careless errors can occur due to carelessness of course, but frequently you will find that you made these kind of mistakes if you did not read the directions properly, skipped part of the problem, calculated wrong, or were rushing. If you find you made a lot of careless errors, make a note to yourself that next time you will have to:

read the directions carefully;
plan your time so you don't have to rush;
work the easier problems first; and,
verify your answer by working the problem backwards or estimating.

Knowledge errors involve a lack of knowledge. Sometimes you simply didn't study enough, but perhaps you learned the topic incorrectly, understood only part of the topic, or studied the wrong topics. Knowledge errors can also occur when you forget to review your text and notes and so fail to understand how all the objectives work together. Study errors can be reduced by:

doing all your assigned practice work (and assigning yourself even more, if necessary);
checking your answers when practicing;
using tutors and coming to the professor's office hours to ask questions;
taking notes and reviewing them before the test; and
reviewing old material every week or two so that you don't start forgetting it.

Application errors are the most difficult to overcome, because it isn't the math that's holding you back -- it's an understanding of how words can communicate math problems. Application errors often occur because you haven't seen a problem worded in a particular way before, because the problem uses words with a special meaning in mathematics, or because you don't notice the pattern or relationship in the problem. To reduce the number of application errors you make:

do as many application problems for practice as possible;
learn the meaning of mathematical words;
read the questions completely;
visualize the problem and draw diagrams or pictures;
make a plan by asking yourself "What do I need to know?", and "What do I have to do to get that knowledge?";
estimate what a reasonable answer might look like before you actually start the problem; and,
check that your answer makes sense.

Once you have analyzed what kinds of mistakes you make on which topics you can take action to stop yorself from repeating the same mistakes. Take part of your study time each week to review and to practice past topics and to connect them to the topics on which you are currently working. Most imporantly, don't assume a problem is going to go away if you do nothing! People tend to repeat their errors unless they actively work to reduce them, and the topics you missed will almost certainly be on the final and will probably be used or developed in your next math course.

Math Analysis Tips Questions

Please answer the following questions in complete sentences. You may use another sheet of paper if you wish.

Analyze your test as described in the essay. Make certain you label every error on your test with the section in your text that the problem came from and whether you believe the points were lost due to "miscommunication", or were "careless", "knowledge", or "strategy" errors.

1) Which two sections in the textbook were responsible for the most errors on your test? Why do you think that occurred?

2) How many points were lost due to "miscommunication"? Is there any one thing you could do to reduce this problem?

3) How many points were lost due to each of the three kinds of errors: "careless", "knowledge", "strategy"? State each category's point loss separately.

4) In problem #3, you stated the point loss for each kind of error. Take the type of error which lost you the most points and reread the advice for correcting it on page 81. Do you think you will try the advice in the essay? Why or why not?

LERN 48

Second Part

Draft Edition

by Eric Kaljumägi

Date: ____________

A note to students:

Hello! By the time you open this book you should be about halfway through your LERN 48 course. Make sure you keep the textbook you have used so far -- chances are that it will be used in your LERN 49 course.

For the remainder of the semester, you will be using this text, which introduces you to the same topics you will see in the beginning of LERN 49, but without as much detail. You will notice that the entire right half of each page is blank save for a vertical line. This is intended for your notes. A good idea is to place the examples your professor gives you in one column and to use the other column for your own comments. If you keep each page of notes aligned with the objective on the other side of the page, you will have created a useful study tool.

Table of Contents:

1

Finding the Natural Factors of a Nnumber

Date: ____________

Definitions: factors -- numbers that are multiplied together
divisor -- a number that divides a given number exactly
(without a remainder)

Factors can be visualized by using objects to make rectangular shapes (called arrays). For example, if we had fourteen objects, we could make a single row of the fourteen,... or two rows of seven,

00000000000000

0000000
0000000

but three rows wouldn't make a rectangle.

00000
00000 <-- three rows, but they aren't even
0000

Thus 1 and 2 are factors of 14, but 3 is not a factor of 14. Since multiplication is commutative, we could have made fourteen rows of one or seven rows of two, so 7 and 14 are also factors of 14. It turns out that 1, 2, 7, and 14 are the only factors of 14.

To find the factors of a given number:

1) **Make a list of numbers from one up to the number which when squared is at least as large as the given number.**
2) **Test each number on the list to see if it is a divisor of the given number.**
3) **If the number is a divisor, then it is a factor. The given number divided by the factor is also a factor.**

Ex.: Find all the factors of 20.

1 x 20
2 x 10
~~3~~ *(20 ÷ 3 has a remainder)*
4 x 5
(We can stop at 4 since the next number (5) has a square which is larger than 20. That is, $5^2 = 25$, which is larger than 20.)
The factors of 20 are 1, 2, 4, 5, 10, and 20.

2 Multiple and Divisible

Date: ____________

Definitions: <u>multiple</u> -- the product of a natural number and a given number
<u>divisible</u> -- a number that can be divided by a given number
(without leaving a remainder)

The words "multiple" and "divisible" refer to the same numbers, but "multiple" is used when multiplying is being discussed, and "divisible" is used when dividing is being discussed.

To find some multiples of a given number, multiply the given number by 1, 2, 3, 4, ... as needed.

To see if a number is a multiple of another, check whether it is divisible. That is, determine if the remainder upon division is zero or not.

Ex.: Find the first five multiple of 6.

6 x 1 = 6
6 x 2 = 12
6 x 3 = 18
6 x 4 = 24
6 x 5 = 30

Ex.: Is 72 a multiple of 5?

No, since 72 ÷ 5 has a remainder of 2.

$$\begin{array}{r} 14 \\ 5\,\overline{)72} \\ \underline{-5} \\ 22 \\ \underline{-20} \\ 2 \end{array}$$

Ex.: Is 84 divisible by 4?

Yes, since 84 ÷ 4 has a remainder of 0.

$$\begin{array}{r} 21 \\ 4\,\overline{)84} \\ \underline{-8} \\ 04 \\ \underline{-4} \\ 0 \end{array}$$

3 Prime and Composite

Date: ____________

Definitions: <u>prime</u> -- a natural number with exactly two natural factors
<u>composite</u> -- a natural number with more than two natural factors.

If you look back at objective #1 the example shows that 20 has six factors (1, 2, 4, 5, 10, and 20). Therefore, 20 is a composite number. Since objective #1 gives a perfectly good way to find factors, we will just modify it slightly here.

To prove whether a natural number is prime or composite:

1) **Use the technique from objective #1 to find the factors of the number.**
2) **If a factor other than one or the number itself is found, the number is composite. If not, it is prime.**

Ex.: Is 34 prime or composite?

1 x 34
2 x 17
(We can stop checking, since we have found a factor besides 1 and 34.)
3
4
5
(If we were looking for all the factors, we could stop at 5 since the next number (6) has a square which is larger than 34. That is, $6^2 = 36$, which is larger than 34.)
34 is a composite number.

Ex.: Is 37 prime or composite?

1 x 37
~~2~~
~~3~~
~~4~~
~~5~~
6
(We can stop at 6 since the next number (7) has a square which is larger than 37. That is, $7^2 = 49$, which is larger than 37.)
37 is a prime number.

Note: 0 and 1 are neither prime nor composite. This is because 0 is not a natural (counting) number and because 1 has only one natural factor, not the two required for primeness.

4 Prime Factorization

Date: ____________

Definitions: factorization -- a number written as a multiplication problem
prime factorization -- a number written as a multiplication problem consisting of only prime factors

A factorization is simply a multiplication problem. Here are several factorizations of 20:

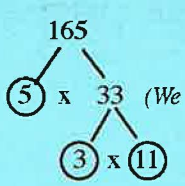
1 x 20 2 x 10 1 x 4 x 5 2 x 2 x 5

However, the last factorization is of particular interest to us, since it only uses prime numbers. This will be helpful later. It so happens that every natural number has only one factorization that uses only prime factors (if you don't count different arrangements of the same prime numbers like 2 x 5 x 2). This is so important that it is called the "fundamental property of arithmetic".

To write the prime factorization of a number using the "factor tree" method:

1) **Factor the desired number in any way that does not use "1".**
2) **If any factor is a prime number, circle it.**
3) **If any factor is not a prime number, repeat these steps with the factor.**
4) **Write down a multiplication problem using the circled factors as your answer.**

Ex.: Find the prime factorization of 165.



(We could have started with 3 x 55 or 11 x 15 -- they all work)

Thus, the prime factorization of 165 is 3 x 5 x 11.

Note: The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ... However, over 90% of our work with prime numbers just involves the first five. You should memorize the following statement:

"The first five primes are 2, 3, 5, 7, and 11."

5 Divisibility Shortcuts for 2, 5, 10, 3, & 9

Date: ____________

We will need to check whether one number is divisible by another quite frequently in this course. Using long division to check whether or not the remainder is zero can become rather tedious. Fortunately, there are several shortcuts one can use. We will only address two of these here, saving the others for later.

Divisibilty Test #1 (look at the last digit on the right):

A number is divisible by 2 if the last digit is 0, 2, 4, 6, or 8.
A number is divisible by 5 if the last digit is 0 or 5.
A number is divisible by 10 if the last digit is 0.

Divisibility Test #2

A number is divisible by 3 if the digit sum is a multiple of 3.
A number is divisible by 9 if the digit sum is a multiple of 9.

Ex.: Is 81,346 divisible by 2?
Yes, since the last digit is a "6".

Ex.: Is 81,347 divisible by 3?
No, since the sum 8 + 1 + 3 + 4 + 7 = 23, and 23 is not a multiple of 3.

Ex.: Is 81, 348 divisible by 3?
Yes, since the sum 8 + 1 + 3 + 4 + 8 = 24, and 24 is a multiple of 3.

Ex.: Is 81,348 divisible by 5?
No, since the last digit is an "8" (not a 0 or 5).

Ex.: Is 81,349 divisible by 9?
No, since the sum 8 + 1 + 3 + 4 + 9 = 25, and 25 is not a multiple of 9.

Ex.: Is 81,350 divisible by 10?
Yes, since the last digit is a "0".

Note: If you know other divisibility shortcuts, ask your professor if you may use them in this class. Don't forget that you can always tell if a number is divisible by dividing. If the remainder is zero, the number is divisible. If the remainder is not zero, the number is not divisible.

6 Writing Fraction Notation For a Set of Objects

Date: ____________

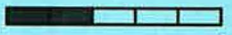
Definitions: fraction -- a division problem
numerator -- the top number (dividend) of a fraction
denominator -- the bottom number (divisor) of a fraction

We often think of fractions as:

$$\frac{\text{number of parts with the quality we want}}{\text{number of parts needed to make one whole group}}$$

Keep in mind that the parts must be equal in size!

Ex.: What fraction of the rectangle is shaded?

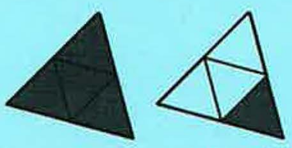


Since 2 parts are shaded, and it takes 5 parts to make one whole rectangle, the fraction is $\frac{2}{5}$.

Ex.: What fraction of the circle is shaded?

Since 3 parts are shaded, and it takes 8 parts to make one whole circle, the fraction is $\frac{3}{8}$.

Ex.: What fraction of the diagrams below are shaded?



Since 5 parts (the small triangles) are shaded, and it takes 4 parts to make one whole triangle, the fraction is $\frac{5}{4}$.

7 Adding like fractions

Date: ____________

Definitions: <u>like fractions</u> -- fractions that have the same denominator
<u>unlike fractions</u> -- fractions that have different denominators

One can only add or subtract <u>like</u> things.

To illustrate this, consider adding twenty pesos and thirty pesos. This is relatively straightforward (the answer is fifty pesos). However, if we add twenty pesos and thirty euros, the problem is much harder since pesos and euros are not the same kind of money. (The answer by the way, is about 495 pesos at the time of this writing.)

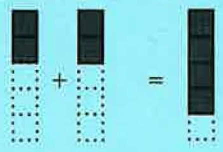
When we work with fractions, the attribute that defines them is their denominator. If the denominators are the same, an addition problem is very much like adding pesos and pesos. If the denominators are not the same, we will have more work to do. For the time being, we will only worry about when the denominators are the same. That is, we will only work with like fractions.

To add like fractions:

1) **Add the numerators of the fractions**
2) **Write the total you found in step 1) as the numerator of the answer**
3) **Write the common denominator as the denominator of the answer**
4) **Simplify, if possible (We will learn how to do this later.)**

Ex.: $\frac{2}{5}+\frac{2}{5}=\frac{4}{5}$

(To see this, picture it visually:



2 + 2 = 4, where we are counting with shaded boxes called "fifths".)

Ex.: $\frac{7}{9}+\frac{1}{9}=\frac{8}{9}$

(Notice that the denominator <u>does not change</u> when adding.

8 Subtracting like fractions

Date: ____________

Once again, one can only add or subtract like things. Statements like "twenty minus one equals one" seem to be incorrect, but twenty cups minus one quart does equal one gallon. The math facts we know only work when the things we are working wih are like each other, and fractions are like when they share the same denominator.

To subtract like fractions:

1) **Subtract the numerators of the fractions**
2) **Write the difference you found in step 1) as the numerator of the answer**
3) **Write the common denominator as the denominator of the answer**
4) **Simplify, if possible (We will learn how to do this later.)**

Ex.: $\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$

(To see this, picture it visually:

5 – 2 = 3, where we are counting with shaded slices called "eighths".)

Ex.: $\frac{5}{6} - \frac{4}{6} = \frac{1}{6}$

(Notice that the denominator does not change when subtracting.)

9 Special Fractions

Date: ______________

Remember that we often think of fractions as:

$$\frac{\text{number of parts with the quality we want}}{\text{number of parts needed to make one whole group}}$$

Based on this idea, there are four types of special fractions that are very important.

$\frac{n}{n} = 1$, where n is any number besides zero

Since the numerator is the number of parts we have, and the denominator is the number of parts needed to make one whole group, if they are the same then you have exactly as many parts as are needed to make one whole! (However, if you try this with zero you find that it takes no parts to make something, which is too strange to contempate for long!)

Ex.: $\frac{6}{6} = 1$

$\frac{n}{1} = n$, where n is any number

If you examine the denominator, it tells you that one part is needed to make a whole group. In other words, each "part" is really a whole object. Since the numerator tells the number of parts, in this instance it also tells the number of whole objects.

$\frac{n}{0}$ is undefined, where n is any number

Here the denominator tells you that it takes no parts to make a whole group, but you have some parts. How many whole groups can you make? This is a logical headache better left unpondered.

$\frac{0}{n} = 0$, where n is any number besides zero

This time the denominator tells you that it takes "n" parts (whatever "n" is) to make a whole group, but you have no parts. Therefore, you can make nothing. (If n = 0, it would appear that both the first and this last rule on this page would apply, which is a good reason to call $\frac{0}{0}$ undefined.)

Ex.: $\frac{0}{6} = 0$

10 Whole Number Multiples of Fractions

Date: ______________

Definition: multiple -- the product of a natural number and a given number (this is repeated from objective #2)

To review, here's how to add like fractions:

1) **Add the numerators of the fractions**
2) **Write the total you found in step 1) as the numerator of the answer**
3) **Write the common denominator as the denominator of the answer**
4) **Simplify, if possible (We will learn how to do this later.)**

Also to review, to find some multiples of a given number, multiply the given number by 1, 2, 3, 4, ... as needed.

This "given number" doesn't have to be a whole number. To see how one could find multiples of fractions, it is helpful to remember that multiplication is repeated addition. For example,

3 x 6 is the same as 6 + 6 + 6 (3 times we have 6 for a total of 18).

Therefore,

$$3 \text{ x } \frac{5}{6} \text{ is the same as } \frac{5}{6} + \frac{5}{6} + \frac{5}{6}$$

$$= \frac{15}{6}$$

If we remember that the special fraction $\frac{3}{1} = 3$, we can then see that

$$3 \text{ x } \frac{5}{6} = \frac{3}{1} \text{ x } \frac{5}{6}$$

$$= \frac{15}{6}$$

If you notice a pattern here, you may use it on your assignment. Otherwise, just use repeated addition for now. (We'll make a "recipe" for this later.)

Ex.: Find 2 x $\frac{6}{11}$.

$$2 \text{ x } \frac{6}{11} = \frac{6}{11} + \frac{6}{11}$$

$$= \frac{12}{11}$$

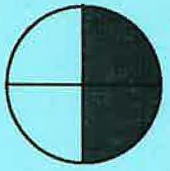
11 Equivalent Fractions

Date: ______________

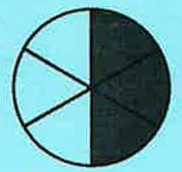
Definition: equivalent fractions -- fractions that represent the same amount

Think about your pocket change for a moment. If you have ten nickels, you have 50 cents. Five dimes also are 50 cents, and two quarters are the same amount as well. Yet if you just look at them, ten nickels and two quarters appear very different from one another. Our coins form fractions of a dollar, and fractions of all types share this idea of "equivalency". That is, fractions that appear different can actually represent the same amount.

To see this visually, look at the following diagrams:



$\frac{2}{4}$



$\frac{3}{6}$

$\frac{4}{8}$

These all are different looking fractions, but in each case you have half of the circle shaded. Thus, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{4}{8}$ are equivalent fractions.

The reason for this has to do with the fact that $\frac{n}{n} = 1$. As you know, multiplying by one does not change a number. (For example, 6 x 1 = 6.) However, where fractions are concerned, multiplying by one can change the appearance of the fraction. This is explained further in the example below.

To make an equivalent fraction given the new denominator:

1) **Find what you have to multiply the first denominator by to equal the second (new) denominator.**
2) **Multiply the first numerator by the value you found in step 1).**

Ex.: Find a fraction equivalent to $\frac{6}{8}$ with a denominator of 24.

$\frac{6}{8} \cdot 1 = \frac{x}{24}$ (since multiplying by one doesn't change the value)

$\frac{6}{8} \cdot \frac{3}{3} = \frac{x}{24}$ (since 8 times 3 is what equals 24)

$\frac{6}{8} = \frac{18}{24}$

(it is o.k. to leave this line out of your work)

12 Simplifying Fractions

Date: ____________

Definition: <u>completely simplified fraction</u> -- a fraction whose numerator and denominator have no common factors other than one

(This section will make use of prime factorization (objective #4). If you have forgotten how to find a prime factorization, be sure to review it before reading on.)

It is possible for two people to both work a fraction problem correctly but yet come up with very different looking answers. This can be confusing. To make sure each correct answer <u>looks the same</u> we need to make sure the answer is completely simplified. Two techniques for doing this are below:

To simplify fractions:

Method 1 (prime factorization technique):

1) **Write both the numerator and denominator in prime factorization form.**
2) **Cancel any factors that are identical in both the numerator and denominator.**
3) **Multiply together the remaining factors in the numerator to get the numerator's answer. Do the same for the denominator.**

Note: If all factors cancel, write "1", not "0"!

Method 2 (traditional cancelling technique):

1) **Find any factor that is common to both the numerator and the denominator.**
2) **Divide both the numerator and denominator by the factor you found in part 1) to get a partially reduced fraction.**
3) **Repeat steps 1) and 2) until the numerator and denominator have no factors in common besides one.**

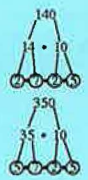
Method 2 is easier when working with common fractions. However, method 1 is considerably more powerful and will be needed for Algebra.

Ex.: Simplify $\frac{140}{350}$.

<u>Method 1:</u>

$$\frac{140}{350} = \frac{\cancel{2}\cdot 2\cdot\cancel{5}\cdot\cancel{7}}{\cancel{2}\cdot 5\cdot\cancel{5}\cdot\cancel{7}}$$

$$= \frac{2}{5}$$



<u>Method 2:</u>

$$\frac{140}{350} \div \frac{10}{10} = \frac{14}{35}$$

$$\frac{14}{35} \div \frac{7}{7} = \frac{2}{5}$$

13 Multiplying Fractions

Date: ____________

Back in objective #10 we saw the following example:

$$3 \text{ x } \frac{5}{6} = \frac{3}{1} \text{ x } \frac{5}{6}$$

$$= \frac{15}{6}$$

Could multiplying fractions be as simple as multiplying across the problem? For the most part, the answer is yes.

To multiply fractions and mixed numbers:

1) **Convert each number to fraction form.**
2) **Simplify by cancelling any factors that are identical in both the numerator and the denominator.**
3) **Multiply the remaining factors in the numerator to obtain the numerator of the answer. Do the same for the denominator.**
4) **Completely simplify the answer.**

Ex.: Find $\frac{14}{30} \bullet \frac{3}{70}$.

$$\frac{14}{30} \bullet \frac{3}{70} = \frac{\cancel{2} \bullet \cancel{7}}{\cancel{2} \bullet \cancel{3} \bullet 5} \bullet \frac{\cancel{3}}{2 \bullet 5 \bullet \cancel{7}}$$ (notice that only top/bottom pairs may be cancelled)

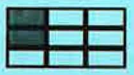
$$= \frac{1}{5 \bullet 2 \bullet 5}$$ ←(if all factors cancel, write "1", not "0")

$$= \frac{1}{50}$$

To see visually how multiplication of fractions works, consider $\frac{1}{3} \bullet \frac{2}{3}$. The word "of" hints at multiplication, so this problem could have been written as "What is one-third of two-thirds?"

<-- Here we have shaded two-thirds of the rectangle

<-- Lightly shaded is one-third of the two-thirds



<-- Which is $\frac{2}{9}$ of the entire rectangle

Thus $\frac{1}{3} \bullet \frac{2}{3} = \frac{2}{9}$.

14 Reciprocals

Date: ____________

Definition: reciprocal (practical definition) -- an operation on a fraction that makes the numerator the denominator and vice-versa.

Reciprocals (also called the multiplicative inverse) are formally defined as the number, which when multiplied by a given number, gives a product of 1. This definition means that all non-zero numbers have a reciprocal. However, we will only worry about the reciprocals of fractions and whole numbers right now.

To find the reciprocal of a number:

1) **Convert each number to fraction form.**
2) **Switch the numerator with the denominator and vice-versa.**

Ex.: Find the reciprocal of $\frac{14}{15}$.

Since this is already in fraction form, the reciprocal is $\frac{15}{14}$.

Ex.: Find the reciprocal of $\frac{1}{4}$.

Since this is already in fraction form, the reciprocal is $\frac{4}{1}$. This simplifies to 4.

Ex.: Find the reciprocal of 7.

7 is not a fraction, but it can be changed to $\frac{7}{1}$, which is a fraction. Switching the numerator with the denominator gives us the reciprocal: $\frac{1}{7}$.

Ex.: Find the reciprocal of 1.

1 is not a fraction, but $\frac{1}{1}$ is. Thus the reciprocal is $\frac{1}{1}$, or 1.

Note: Because finding the reciprocal is usually fairly easy, some students omit a work step and just write $\frac{2}{3} = \frac{3}{2}$. This is not correct!

It is permissible to write $\frac{2}{3} \rightarrow \frac{3}{2}$, but do not use an equal sign when finding reciprocals.

15 Dividing Fractions

Date: ____________

It turns out that although there are techniques for dividing fractions, one can convert any fractional division problem into a multiplication problem as noted below. As a result, few people bother learning a separate technique.

To divide fractions and mixed numbers:

1) **Convert each number to fraction form.**
2) **Convert the problem to a multiplication problem switching the divisor (the second fraction) with its reciprocal.**
3) **Simplify by cancelling any factors that are identical in both the numerator and the denominator.**
4) **Multiply the remaining factors in the numerator to obtain the numerator of the answer. Do the same for the denominator.**
5) **Completely simplify the answer.**

Note that only the second step differs from what was given in objective #13.

Ex.: Find $\frac{12}{30} \div \frac{3}{40}$.

$\frac{12}{30} \div \frac{3}{40} = \frac{12}{30} \bullet \frac{40}{3}$ (note that only the 2nd fraction is flipped)

$$= \frac{\cancel{2} \bullet 2 \bullet \cancel{3}}{\cancel{2} \bullet \cancel{3} \bullet \cancel{5}} \bullet \frac{2 \bullet 2 \bullet 2 \bullet \cancel{5}}{3}$$

$$= \frac{2 \bullet 2 \bullet 2 \bullet 2}{3}$$

$$= \frac{16}{3}$$

To see why this works, note that since fractions are themselves division problems, we can write the division problem as a giant fraction times 1.

$\dfrac{\frac{12}{30}}{\frac{3}{40}} \bullet 1$ Now, replace "1" with $\frac{40}{3}$ over itself.

$= \dfrac{\frac{12}{30}}{\frac{3}{40}} \bullet \dfrac{\frac{40}{3}}{\frac{40}{3}}$ Since $\frac{3}{40}$ and $\frac{40}{3}$ are reciprocals, their product is 1.

$= \dfrac{\frac{12}{30} \bullet \frac{40}{3}}{1}$ All that's left is a multiplication problem.

16 Comparing Two Fractions

Date: ____________

It is easy to compare like things.

Fourteen pesos are more than ten pesos, and six kilograms are less than seven kilograms. Unlike things are much harder to compare. Fourteen pesos is much less than ten dollars, and six kilograms is far more than seven pounds. If things are like each other, the numbers can be compared directly, but if that isn't the case, the comparison is much harder. Fortunately for fractions, there is the "means - extremes" rule (which some people call "cross-products").

To compare two fractions:

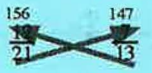
1) **Write the fractions side-by-side.**
2) **Multiply the left denominator by the right numerator and write the product over the right fraction.**
3) **Multiply the right denominator by the left numerator and write the product over the left fraction.**
4) **Whichever fraction has the greater product is the greater fraction.**

Ex.: Which is larger, $\frac{5}{8}$ or $\frac{2}{3}$?

Since 15 < 16,

$\frac{5}{8} < \frac{2}{3}$.

Ex.: Which is larger, $\frac{12}{21}$ or $\frac{7}{13}$?



Since 156 > 147,

$\frac{12}{21} > \frac{7}{13}$.

The reason why this works is because you are actually finding like fractions (fractions with the same denominator).

$\frac{12}{21}$ happens to equal $\frac{156}{273}$, and $\frac{7}{13}$ equals $\frac{147}{273}$.

Since we now have fractions that are like each other, comparing $\frac{156}{273}$ and $\frac{147}{273}$ is very much like comparing 147 pesos and 156 pesos.

We will see another way to make like fractions later.

17 Least Common Multiple (L.C.M.)

Date: ______________

Definition: <u>Least Common Multiple (L.C.M.)</u>: The smallest number that is a multiple of each of two or more given numbers.

(This section will make use of prime factorization (objective #4). If you have forgotten how to find a prime factorization, be sure to review it before reading on.)

The Least Common Multiple is just what its name implies: it is the *smallest* (or least) *multiple* that a set of numbers have *in common*. Finding it might seem to be a useless exercise, but as it happens, the L.C.M. will be needed later.

It is possible to find L.C.Ms by inspection, but this can be tedious.

Ex. Find the L.C.M. of 8, 12, and 15.

the multiples of 8 are 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, ...
the multiples of 12 are 12, 24, 36, 48, 60, 72, 84, 96, 108, 120, ...
the multiples of 15 are 15, 30, 45, 60, 75, 90, 105, 120, 135, ...

Do you see the answer? Perhaps we didn't make the lists long enough. Clearly though, we need a better method, like the one below.

To find the L.C.M. of a set of numbers:

1) **Find the prime factorization of each number.**
2) **Find the prime factorization with the <u>most</u> 2s and write down that many 2s.**
3) **Repeat step 2) with 3, 5, 7, and any other prime numbers that show up in the prime factorizations.**
4) **Multiply together the prime numbers you wrote down.**

Ex.: Find the L.C.M. of 8, 12, and 15.

$8 = \underline{2} \bullet \underline{2} \bullet \underline{2}$ <-- First, we find each prime factorization, and then we write down the most of each type of prime

$12 = 2 \bullet 2 \bullet \underline{3}$

$15 = \underline{3} \bullet \underline{5}$

(in case of a tie, use either number, not both)

L.C.M. $= 2 \bullet 2 \bullet 2 \bullet 3 \bullet 5$

$= 120$

18 Using the L.C.M. to Convert to Like Fractions

Date: ____________

Like things are easier to compare, add, and subtract than unlike things. Fractions are "like" if their denominators are the same. If the fractions are "unlike", it is possible to convert them to "like" fractions using the tools we're practiced earlier in the semester.

To convert fractions to equivalent like fractions:

1) **Find the L.C.M. of the denominators.**
2) **Find what you have to multiply the first denominator by to equal the L.C.M..**
3) **Multiply the first numerator by the value you found in step 2).**
4) **Repeat steps 2) & 3) with the second fraction.**

Ex.: Convert $\frac{5}{8}$ and $\frac{2}{3}$ into equivalent like fractions.

$8 = 2 \bullet 2 \bullet 2$
$3 = 3$

$\text{L.C.M.} = 2 \bullet 2 \bullet 2 \bullet 3$
$= 24$ <-- First, we find the L.C.M. of the denominators

$\frac{5}{8} \bullet 1 = \frac{}{24}$
$\frac{2}{3} \bullet 1 = \frac{}{24}$

<-- Then we find what will make equivalent fractions

$\frac{5}{8} \bullet \frac{3}{3} = \frac{15}{24}$

$\frac{2}{3} \bullet \frac{8}{8} = \frac{16}{24}$

The equivalent fractions are $\frac{15}{24}$ and $\frac{16}{24}$.

1 Finding the Natural Factors of a Number - Worksheet #1, p.1

Vocabulary

2 X 3 = 6 → **product** or **multiple** of 2 or 3

factors

$$\begin{array}{r} 3 \\ 2\overline{)6} \\ \underline{6} \\ 0 \end{array}$$

divisor (goes in evenly; no remainder)

1 X 7 = 7 → **prime**

(No other factors except 1 and itself)

Example

List all the factors of 20.

1 X 20
2 X 10
~~3 X ?~~ (3 does not go into 20 evenly so 3 is not a factor of 20)
4 X 5
~~5 X 4~~ (Stop at 5 because it's square, $5^2 = 5 \times 5 = 25$, is bigger than 20)

Factors of 20 = 1, 2, 4, 5, 10, 20

(Your answer should look like this, with all the factors listed in order.)

1a. Find the **squares** of the following numbers.

1 x 1 =
2 x 2 =
3 x 3 =
4 x 4 =
5 x 5 =
6 x 6 =
7 x 7 =
8 x 8 =
9 x 9 =
10 x 10 =
11 x 11 =
12 x 12 =
13 x 13 =
14 x 14 =

Hint: You may refer back to 1a. to help you find what factor to stop at.

1b. Write the labels on this problem.

2 x 7 = 14 → ____________ or ____________

____

Hint: See the vocabulary above.

1c. List all the factors of 64. Circle the number you will stop at because it's square is greater or equal to 64.

1c. Factors of 64 =
1,2.4,8,16,32,64

1 x 64
2 x 32
4 x 16
⑧ x 8

$8^2 = 64$, so we will stop at 8

1d. List all the factors of 23. Write the word Prime next to your answer if 23 only has 1 and itself as factors.

1d. Factors of 23 =
1,23 Prime

Cover these answers. No peeking! ↑

1

Finding the Natural Factors of a Number - Worksheet #1 p.2

2a. Suppose a number is a **multiple** of four. $4 \times 2 = \boxed{8}$ $4 \times 3 = \boxed{12}$ $4 \times 4 = \boxed{16}$ → **multiples** Does two have to be a **factor** of the number? Why?	2a. Yes, because all of 4 multiples are even, and 2 is a factor of all even numbers.
2b. List all the factors of 21 then list all the factors of 32. Does 21 and 32 share any factors?	2b. Factors of 21 = 1,3,7,21 Factors of 32 = 1,2, 4, 7, 16, 32 21 and 32 share the factors of 1, and 7
3a. List all the factors of 144. Circle the factor you will stop at because its square is greater or equal to 144.	3a. Factors of 144 = 1,2,3,4,6,8,9,12, 16, 18, 24, 36, 48, 72, 144 Stop at 12 because 12 x 12 = 144 which is equal to 144
3b. List all the factors of 52. Circle the factor you will stop at because its square is greater or equal to 52.	3b. Factors of 52 = 1, 2,4,13,16,52 Stop at 8 because 8x8 = 64 which is greater than 52.
3c. List all the factors of 36. Circle the factor will you stop at because its square is greater or equal to 36.	3c. Factors of 36 = 1,2,3,4,6,9,12,18,36 Stop at 6 because 6x6 = 36 which is equal to 36. Cover these answers. ↑ No peeking!

1

Finding the Natural Factors of a Number - Worksheet #1 p.3

4a. List all the factors of 40

4b. List all the factors of 75

4a. Factors of 40 = 1, 2, 4, 5, 8, 10, 20, 40

4b. Factors of 75 = 1, 3, 5, 15, 25, 75

5a. List all the factors of 33.

5b. List all the factors of 31

5a. Factors of 33 = 1, 3, 11, 33

5b. Factors of 31 = 1, 31
31 is Prime

6a. List all the factors of 10

6b. List all the factors of 12

6a. Factors of 10 = 1, 2, 5 10

6b. Factors of 12 = 1, 2, 3, 6, 12

6c. List all the factors of 25

6d. List all the factors of 20

6c. Factors of 25 = 1,5, 25

6d. Factors of 20 = 1,2, 4,5,10,20

7. Simplify this equation by using the proper **Order of Operations**.

$49 \div 7 - (2 + 3) =$

7. 2
Hint: PEMDAS

Cover these answers. ↑
No peeking!

Finding the Natural Factors of a Number - Worksheet #1 p.4 - Review

8a. Solve this equation by using **Order of Operations**.

$12 - 3 + 5 =$

8b. Solve this equation by using **Order of Operations**.

$12 - (3 + 5) =$

8a. 14

8b. 4

Hint: PEMDAS

8c. Write these equations in **exponential** form and solve.

$2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 =$

$5 \bullet 5 \bullet 5 =$

8d. Solve by using **Order of Operations**. Show all steps.

$12 - 3^2 + (5 + 2) - 2^3 =$

8c. $2^6 = 64$
$5^3 = 125$

8d. 2

9a. Solve $\frac{5764}{2}$

$2\overline{)5764}$

9b. Solve $506 \div 12 =$

9a. 2882

9b. 42 r.2

10a. Find the **average** price.

$32.50
$62.20
$50.50
$17.80

10b. Find this student's **grade point average**.

A = 4, B = 3, C = 2, D = 1, F = 0

A =
C =
B =
F =

*F's and D's can really drag your GPA down.

10a. $40.75

10b. 2.25

Cover these answers. No peeking! ↑

2 Multiple and Divisible - Worksheet #2, p.1

Vocabulary

2 x 3 = 6 ⟶ **product** or **multiple** of 2 and 3

factors

6 ÷ 2 = 3

6 is **divisible** by 2 because
2 goes into 6 evenly.

Example

Find the first 5 **multiples** of 5.

5 x 1 = 5
5 x 2 = 10
5 x 3 = 15
5 x 4 = 20
5 x 5 = 25

Is 20 **divisible** by 5?
Yes, because 5 divides into 25
evenly with no remainder.

$$\begin{array}{r} 4 \\ 5\overline{)20} \\ -20 \\ \hline 0 \end{array}$$

1a. Find the first 5 **multiples** of 7.

1a. 7 x 1 = 7
7 x 2 = 14
7 x 3 = 21
7 x 4 = 28
7 x 5 = 25

1b.

Is 18 **divisible** by 9? ______ 18 ÷ 9 = ______ Is there a remainder? ______
Is 18 **divisible** by 4? ______ 18 ÷ 4 = ______ Is there a remainder? ______
Is 18 a **multiple** of 9? ______
Is 18 a **multiple** of 4? ______
Is 9 a **factor** of 18? ______
Is 4 a **factor** of 18? ______

1b.
yes, 2, no
no, 4^{r2}, yes 2
yes 9 x 2 = 18
no
yes
no

1c.

(Show all your work here.)

Is 84 **divisible** by 4? ______
Is 84 **divisible** by 3? ______
Is 84 a **multiple** of 4? ______
Is 84 a **multiple** of 3? ______
Is 4 a **factor** of 84? ______
Is 3 a **factor** of 84? ______

1c. yes, 84 ÷ 4 = 21
yes, 84 ÷ 3 = 28
yes, 4 x 21 = 84
yes, 3 x 28 = 84
yes
yes

Cover these answers. ↑
No peeking!

2 Multiple and Divisible - Worksheet #2, p.2

2a. List the first 10 **multiples** of 3, and the first 10 **multiples** of 6.
Circle the **common multiples** that are on the 3 list and the 6 list.

2a.	X3	X6
X1	3	⑥
X2	⑥	⑫
X3	9	⑱
X4	⑫	㉔
X5	15	30
X6	⑱	36
X7	21	42
X8	㉔	48
X9	27	54
X10	30	60

2b. If you have a $20.00 bill in your wallet and 3 of your children need lunch money for the week at $2.00 per lunch for 5 days, would you have enough money?

Is $20.00 **divisible** by 3 or $6? _______

How many days of lunch could you buy for your three children,
and how many days this week would you have to make them a bag lunch?

2b.

3 kids
x $2 / lunch
$6 / day

$6 x 1 day = $6
$6 x 2 days = $12
$6 x 3 days = $18
$6 x 4 days = $24
$6 x 5 days = $30

$20.00
- $18.00 for 3 days
$2.00 in change

You would have to make bag lunches for 2 days.

2c. You need to buy bus tickets at a vending machine that does not give back any change. You have a $20.00 bill and $5.00 bill in your wallet. Which button would you push to minimize the amount of change you would loose to the machine while getting the most tickets for your money. You many push any of the buttons more than once.

☐ 1 one way ticket for $2.00

☐ 1 round trip ticket for $3.00

☐ 6 one way tickets for $11.00

☐ 6 round trip tickets for $16.00

2c.
One - 6 round trip
Two - 1 round trip

Cover these answers.
No peeking! ↑

2 Multiple and Divisible - Worksheet #2, p.3

3a. Use the words **multiple**, **divisible**, and **factor** to label these equations.

10 is a __________ of 5 and 2

↑

5 x 2 = 10

\/

5 and 2 are __________ of 10

10 ÷ 2 = 5

↓

10 is __________ by 2

3a.
10 is a multiple of 5 and 2

5 and 2 are factors

10 is divisible by 2

3b. Explain why 10 is not **divisible** by 4.

3b.
10 is not **divisible** by 4 because 4 does not divide into 10 evenly.

3c. Explain why 10 is not a **multiple** of 4.

3c.
10 is not a **multiple** of 4 because 4 times any whole number does not equal 10.

4a.

Is 36 **divisible** by 9? ______ 36 ÷ 9 = ______ Is there a remainder? ______

Is 36 **divisible** by 7? ______ 36 ÷ 7 = ______ Is there a remainder? ______

Is 36 a **multiple** of 9? ______

Is 36 a **multiple** of 7? ______

Is 9 a **factor** of 36? ______

Is 7 a **factor** of 36? ______

4a.

yes, 4, no

no, 5R1, yes 1

yes 9 x 4 = 36

no

yes

no

4b. Fill in the blanks with any whole numbers that would make these sentences correct.

______ is **divisible** by ______.

______ is a **multiple** of ______.

4b.
Answers may vary.

Cover these answers.
No peeking! ↑

Multiple and Divisible - Worksheet #2, p.4

5a. Find the first 5 **multiples** of 7.	5a. 7 14 21 28 35
5b. Find the first 5 **multiples** of 8.	5b. 8 16 24 32 40
5c. Find the first 5 **multiples** of 4 and the first 5 **multiples** of 2. Circle the **common multiples**.	5c. X4 X2 ④ 2 ⑧ ④ 12 6 16 ⑧ 20 10
6a. Is 76 a **multiple** of 2? Explain why.	6a. Yes, because 76 is an even number.
6b. Is 37 a **multiple** of 3? Explain why.	6b. No
6c Is 121 a **multiple** of 11? Explain why.	6c. Yes

Cover these answers.
No peeking! ↑

Multiple and Divisible - Worksheet #2, p.5 - Review

7a. List all the **factors** of 28.

7b. List all the **factors** of 33

7a. Factors of 28 = 6, 2, 4, 7, 14, 28

7b. Factors of 33 = 1, 3, 11, 33

7c. List all the **factors** of 13.

7d. List all the **factors** of 64.

7c. Factors of 13 = 1, 13 Prime

7d. Factors of 64 = 1, 2, 4, 8, 16, 32, 64

8a. Solve this equation by using the proper **Order of Operations**.

$18 - 3^2 + (7 + 3) - 2^2 =$

8b. Solve this equation by using the proper **Order of Operations**.

$(2 + 3)^2 - 5 + 2^3 =$

8a. 15

8b. 28

8c. Solve this equation by using the proper **Order of Operations**.

$34 - 3 + 5 \times 2^2 =$

8d. Solve this equation by using the proper Order **of Operations**.

$34 - (3 + 5) \times 2^2 =$

8c. 51

8d. 2

Cover these answers.
No peeking! ↑

2 Multiple and Divisible - Worksheet #2, p.6 - Review

9a. Use **long division** to solve this problem.

$4\overline{)3062}$

9b. Use **long division** to solve this problem. 464 ÷ 4 =

9a. 765 R2

9b. 116

9c. Use **long division** to solve this problem. 7895 ÷ 5 =

9d. Use **long division** to solve this problem. $\frac{254}{2}$

9c. 1579

9d. 127

10a. Find the **average** price.

$13,00
$45.25
$ 9.00
$10.00
+$17.00

10b. Find the **Grade Point Average** for this student.
A = 4, B = 3, C = 2, D = 1, F = 0

C
D
A
+B

10a. $18.85

10b. GPA = 2.5

10c. Find the **average** price.

.06¢, $1.06, .76¢, .24¢

10d. The Los Angeles Times reported that the **average** price of a home in Los Angeles County was $400,000 in 2007. How did the reporters come up with this **average**?

10c. .53¢

10d. The reporters added all the home prices, then divided the sum by how many homes were listed.

Cover these answers.
No peeking! ↑

3 Prime and Composite - Worksheet #3, p.1

Vocabulary

List the factors of 13.
1 x 13 = 13
Factors of 13 = 1,13
↓
prime number

List all the factors of 15.
1 x 15 = 15
3 x 5 = 15
Factors of 15 = 1, 3, 5, 15
↓
composite number

Note: 0 and 1 are not prime or composite numbers.

Example

Is 26 prime or composite?
1 x 26
2 x 13
~~3~~
~~4~~
~~5~~
~~6~~ (Stop at 6 because it's square $6^2 = 36$, which is greater than 26.)
Factors of 26 = 1, 2, 13, 26
26 is **composite**

1a. Label these equations using the words **factors, product** or **multiple, prime,** and **composite.**

1 x 7 = 7 → ____________ or ____________

____

7 is a ______________ number because it only has one and itself as factors. Nothing can divide into 7 evenly.

1 x 8 = 8
2 x 4 = 8 > ____________ or ____________

____

8 is a ______________ number because it has many factors. The factors 2 and 4 can divide into 8 evenly.

1b. Are all even numbers **composite**. Why?

1c. Are all odd numbers **prime**? Why?

3 Prime and Composite - Worksheet #3, p.2

2a. Make a list of the all prime numbers between 1 and 20.

2a. 3, 5, 7, 11, 13 17, 19.

2b. Eratosthenes was a Greek mathematician who invented a way to find all the **prime numbers** from 1 to 50.

Sieve of Eratosthenes

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Step one: Cross out 1, because 1 is not considered a prime number.
Step two: Circle 2 because it is a prime number, then cross out all the multiples of 2. (All the even numbers)
Step three: Circle 3 because it is prime number, then cross out all the multiples of 3. (6, 9, 12, etc.)
Step four: Circle 5 because it is a prime number, then cross out all the multiples of 5. (Hint: any number ending in 0 or 5)
Step five: Circle 11 because it is a prime number. Now your done! Circle any numbers that are left; they are all prime! Now say Eratosthenes five times real fast to impress your friends.

2b. Follow the steps.

2c. Remember that **composite numbers** are any number greater than 1 that is not prime. A composite number has more than one and itself as factors; other numbers can divide evenly into composite numbers. 6 is an example of a composite number because it has the factors 1 x 6, and 2 x 3.
2 and 3 can both divide into 6 evenly.

List all the numbers from 2 to 50 that are composite.

2c. Hint: look at the Sieve of Eratosthenes.

Cover these answers.
No peeking! ↑

Prime and Composite - Worksheet #3, p.3

Example of a good answer that shows all work.

4a. Is 45 prime or composite?

Factors of 45 =
1 x 45
~~2~~
3 x 15
~~4~~
5 x 9
~~6~~
7 (Stop at 7 because $7^2 = 49 > 45$)

45 is composite

4b. Is 31 prime or composite?

4c. Is 73 prime or composite?

4d. Is 23 prime or composite?

5a. Is 17 prime or composite?

5b. Is 86 prime or composite?

5c. Is 33 prime or composite?

5d. Is 12 prime or composite?

6a. Fill in the blank.

33 is **divisible** by ______.

6b. Fill in the blank.

17 is only **divisible** by _______.

4a. 45 is composite

4b. 31 is prime

4c. 73 is prime

4d. 23 is prime

5a. 17 is prime

5b. 86 is composite

5c. 33 is composite

5d. 12 is composite

6a . 11 or 3

6b. 17

Cover these answers. ↑
No peeking!

Prime and Composite - Worksheet #3, p.4 - Review

7a. Find the first 5 multiples of 6.

7b. Find the first 5 multiples of 2 and the first five **multiples** of 8. Circle the common multiples.

7a. The first 5 multiples of 6 = 6, 12. 18. 24. 30.

7b.

	x2	x8
x1	2	⑧
x2	4	16
x3	6	24
x4	⑧	32
x5	10	40

8a. List all the **factors** of 28.

8b. List all the **factors** of 22.

8s. Factors of 28 = 1, 2, 4, 7, 14, 28

8b. Factors of 22 = 1, 2, 11, 22

9a. Solve this equation by using the proper **Order of Operations**.
$9^2 - (6 - 3) + 5 \times 2 =$

9b. Solve this equation by using the proper **Order of Operations**.
$9^2 - 6 - 3 + 5 \times 2 =$

9a. 88

9b. 82

10a. Use **long division** to solve this problem. 6782 ÷ 16 =

10b. Find the **average** price.

$$\begin{array}{r} \$350 \\ \$450 \\ +\$240 \\ \hline \end{array}$$

10a. 423 R15

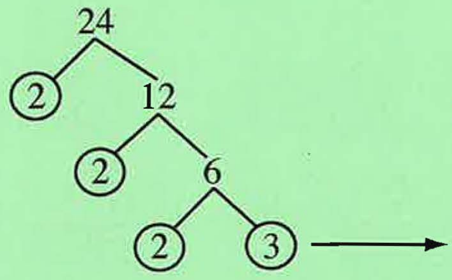
10b. $346.66

Cover these answers.
No peeking! ↑

4 Prime Factorization - Worksheet #4, p.1

Find the **prime factorization** of 124 using the "factor tree" method.

Example



$24 = 2^3$ x 3

All the circled numbers are prime factors.

1a. List all the **prime numbers** from 1 to 14. Memorize this list.

1a. 3, 5, 7, 11, 13

1b. List all the **factors** of 24.

1c. Write the **prime factorization** of 24.

1b. Factors of 24 = 1, 2, 3, 4, 6, 8, 12, 24.

1c. $24 = 2^3$ x 3

1d. What are the differences between the all the **factors** of 24 and the **prime factorization** of 24?

1d. Answers will vary.

2a. All natural numbers have one **prime factorization**. This is called the **"fundamental property of arithmetic."** Fill in the blanks.

Many **factorizations** of 24.

____ x 24 = 24
____ x 12 = 24
____ x 8 = 24
____ x 6 = 24
2 x ____ x 6 = 24

Only one **prime factorization** of 24.

2 x 2 x 2 x ____ = 24

2a. 1, 2, 3, 4, 2, 3.

Cover these answers. No peeking! ↑

4

Prime Factorization - Worksheet #4, p.2

3a. Find the prime factorization of 124.

3b. Find the prime factorization of 64.

3a. The prime factorization of 124 = 2^2 x 31

3b. The prime factorization of 64 = 2^6

4a. Find the prime factorization of 61.

4b.Find the prime factorization of 300.

4a. The prime factorization of 61 = 1 x 61

4b. The prime factorization of 300 = 2^2 x 3 x 5^2

5a. Find the prime factorization of 12.

5b. Find the prime factorization of 35.

5a. The prime factorization of 12 = 2^2 x 3

5b. The prime factorization of 35 = 5 x 7

6a. Find the prime factorization of 72.

6b.Find the prime factorizaation of 49.

6a. The prime factorization of 72 = 2^3 x 3^2

6b. The prime factorization of 49 = 7^2

Cover these answers.
No peeking! ↑

Prime Factorization - Worksheet #4, p.3 - Review

7a. Label this equation with the words **factors, muliple or product, divisible.**

2 x 6 = 12 →

____

____

12 is _____________ by 3 and 2

7b. Is 121 **prime** or **composite**? List all **factors**. (Show all work)

7a.

7b.
121 is composite
List all the factors.
(See section 3)

7c. Is 36 **prime** or **composite**? List all **factors**.

7d. Is 31 **prime** or **composite**? List all **factors**.

7c.
36 is composite

7d.
31 is prime

8a. List the first 5 **multiples** of 7.

8b. Is 72 a **multiple** of 6? (Show work)

8a.
1 x 7 = 7
2 x 7 = 14
3 x 7 = 21
4 x 7 = 28
5 x 7 = 35

8b. Yes

8c. Is 72 **divisible** by 9?

8d. Find the **prime factorization** of 72. Show the "factor tree".

8c. Yes

8d.
The prime factorization of $72 = 2^3 \times 3^2$

Cover these answers. No peeking! ↑

Prime Factorization - Worksheet #4, p.4 - Review

9a. Find all the **factors** of 20.

9b. Find all the **factors** of 34.

9a. Factors of 20 =
1, 2, 4, 5, 10, 20

1 x 20
2 x 10
~~3~~
4 x 5

(Answers should look like this)

9b. Factors of 34 =
1, 2, 17, 34

9c. Find all the **factors** of 64.

9d. Find all the **factors** of 12.

9c. Factors of 64 =
1, 2, 4, 8, 16, 32, 64

9d. Factors of 12 =
1, 2, 3, 4, 6. 12

10a. Solve this equation using the proper **Order of Operations**.

$(2 + 1)^3 - 4^2 + 2 \times 2 =$

10b. Solve this equation using the proper **Order of Operations**.

$(2 + 1)^3 - (4^2 + 2) \times 2 =$

10a. = 15
Hint: PEMDAS

10b. = 18

10c. Find the average price.

$45.30
$25.50
$ 8.00
+ $20.00

10d. Find this student's GPA (Grade Point Average)
A, A, B, D, C, C

10c. The average price = $24.70

10d. GPA = 2.66

Cover these answers.
No peeking! ↑